CHAPTER 6

6.0 SUMMARY AND RECOMMENDATIONS OF THE STUDY

6.1 Introduction

This chapter presents a summary of the study, recommendations, and suggestions for further research. The main aim of this study was to investigate how competent mathematics teachers whose learners perform consistently well in the Grade 12 mathematics National Senior Certificate Examination develops pedagogical content knowledge in school statistics teaching. Specifically, it explored what these teachers do in classroom practice when teaching data handling topics to the learners. In addition, the study probed the implications that PCK has for mathematics teacher education programmes (ref Section 6.5).

6.2 Focus of the study

The study investigated how competent mathematics teachers develop PCK in statistics teaching (which has recently been introduced as a formal aspect of mathematics in the National Curriculum Statements (now changed to Curriculum and Assessment Policy Statements (CAPS)) in South Africa. The chief examiner's report (DoE, 2009; DoBE, 2012) states that learners' poor performance in statistics may mean that mathematics teachers have not acquired sufficient PCK for teaching the subject. In addition, delegates at the International Commission for Mathematics Instruction and International Association for Statistics Educators joint conferences (ICMI/IASE, 2007, 2011) attributed learners' poor achievement in statistics to underdevelopment of PCK by practising mathematics teachers. This research is therefore intended to explore the manner in which competent mathematics teachers develop PCK in statistics teaching.

A multi-method approach involving the use of several research instruments such as a conceptual knowledge exercise, concept mapping, lesson observation, teacher questionnaires, interviews, written reports, video records and document analysis for data collection, was adopted to carry out the investigation. Mathematics teachers were identified who were perceived to be competent in teaching mathematics, based on their school performance in the senior certificate examination, together with recommendations from principals, peers and subject facilitators.

The research questions that guided the study are:

1) What subject matter content knowledge of statistics do the mathematics teachers have and demonstrate during classroom practice?

2) What instructional skill and strategies do these teachers use in teaching statistics?

3) What knowledge of learners' preconceptions and learning difficulties, if any, do they have and demonstrate during classroom practice?

4) How do these teachers develop PCK in statistic teaching?

A qualitative research approach involving the case study method was used to collect data. The data were analysed to determine the teachers' assumed PCK and how they might have developed their PCK profile in statistics teaching. PCK, in the context of this study, was used as a theoretical framework to try to determine how they developed their assumed PCK in statistics teaching (ref Section 1.7). It was defined as 'an amalgam of practising teachers' content knowledge in school statistics; their pedagogical knowledge (instructional skills and strategies) and learners' conceptions and learning difficulties in statistics teaching' (Shulman, 1987 and Ball et al, 2008: 391)

In applying the PCK as a theoretical framework, certain assumptions were made, as indicated in chapter 1, to enable the investigator to proceed with the study. These assumptions are as follows:

- PCK represents a category of knowledge that describes the quality of an expert teacher (Miller, 2006).
- PCK provides a framework that can be used to describe the origin of its critical teacher knowledge, but not all the teachers have the same PCK (Miller, 2006).
- PCK is a constructivist process and, therefore, a continually changing body of knowledge (Miller, 2006).
- PCK can be measured by conceptualising the construct and using multiple assessment techniques, including classroom practice (Hill, 2008).

It is currently a widely accepted belief that PCK represents a category of knowledge needed for a novice teacher to mature into an expert (Miller, 2006). Ball *et al* (2008) described teacher knowledge as an amalgamation of subject matter and pedagogy. The blend of

different forms of teacher knowledge has forced many teacher education programmes to create new pedagogical activities that engage pre-service teachers in terms of the teachers' classroom practice. 'The same vision of how to improve classroom practice has provided a focus on education research; unfortunately PCK remains a category of knowledge that is difficult to isolate and research' (Miller, 2006). However, the teachers' classroom practice in statistics teaching in the context of this study was investigated in a case study using lesson observation to see how they demonstrated their subject matter content knowledge, pedagogical knowledge, and knowledge of learners' conceptions and learning difficulties. Data gathered from lesson observation were triangulated with data collected from concept mapping, teacher questionnaires, interviews, written reports, video records and document analysis in order to determine how the mathematics teachers develop their PCK in statistics teaching for learner performance and classroom practice improvement.

6.3 Summary of the results according to the theoretical framework

A summary of the results from the investigation is as follows:

6.3.1 Knowledge of the subject matter content

The four participating teachers taught statistical graphs predominantly using procedural knowledge and less frequently as conceptual knowledge. The use of procedural knowledge was to some extent dictated by the nature of the topic, which required learners to be able to collect, organise, analyse and interpret statistical and probability models to solve related problem (DoBE, 2010). A second factor that leads to the use of procedural knowledge is the way in which statistical graphs should be constructed, which involves drawing axes, choosing scales, labelling axes, plotting points and joining the lines of best fit. Other processes in developing subject matter content knowledge included the frequent use of mathematics textbooks, CAPS documents, as well as attendance at workshops (ref Appendix xvii).

6.3.2 Pedagogical knowledge (instructional skills and strategies)

Instructional skills are the most specific category of teaching behaviour. They are necessary for procedural purposes and for structuring appropriate learning experiences for learners. In this study, instructional skills and strategies, involving construction skills such as drawing axes, choosing scale, labelling axes, plotting the points and joining the lines of best fit were used in constructing statistical graphs. Instructional strategies such as oral probing

questioning, pre-activities, pre- and post-teaching discussion were used by the individual teachers to determine learners' prior knowledge in statistical graph construction. Checking and marking learners' responses to homework were other assessment strategies that helped to determine learners' prior knowledge in statistical graphs and learning difficulties. Procedural and conceptual approaches were used to describe how to construct statistical graphs such as the bar graphs, box-and-whisker plots, ogives, histograms and scatter plots. Individual and grouped classwork, homework and assignments as well as oral probing questioning were also used to assess how well learners' have understood the lessons on these graphical constructions. An analysis of learners' responses to classwork, homework and assignments was the main assessment strategy that the participating teachers used to identify learners' misconceptions and learning difficulties in statistics teaching. While some learners showed that they had grasped what the teachers taught, a few experienced learning difficulties. Instructional strategies such as the use of extra tutoring, class activities in the form of drill and practice, explanation, examples drawn from familiar situations and post-teaching discussions were used to address learners' misconceptions and learning difficulties in statistical graph construction.

The participating teachers claimed that the instructional skills and strategies used in teaching statistics were developed through formal education and classroom practice (ref Section 4.7.2 and Appendix xvii). The development of instructional skills and strategies varies from teacher to teacher, depending on the topic, feedback from the learners, and the learners' prior knowledge of that topic. The results drawn from lesson observation (ref Sections 4.5.1–4.5.4), interviews and the questionnaires (ref Appendices xvii and xxviii) showed that the participating teachers used topic-specific instructional strategy of providing exercises in statistics in which learners were required to solve problems, while the teachers monitored and guided them (as in classwork), which allowed learners to construct knowledge by themselves, thereby influencing their active participation in the lessons. By using instructional skills such as topic-specific construction skills, and the instructional strategies of oral probing questioning, pre-activities, extra tutoring and class activities and post-teaching discussion, as well as assessment strategies of analysing learners' responses to written works to determine learners' misconceptions and learning difficulties, the participating teachers' may have intensified and broaden their knowledge of the instructional skills and strategies used in teaching school statistics.

6.3.3 Knowledge of learners' preconceptions and learning difficulties

The most notable learning difficulty observed in the lessons of all four teachers was the inability to construct and interpret graphs of grouped data (ref Sections 5.2.1-5.2.4). The main challenge, in part, was owing to learners' inability to choose an appropriate scale (ref Sections 5.2.1- 5.2.4). Second, the learners had difficulty in labelling the axes without proper scaling for constructing the statistical graph on the paper provided (ref Figure 4.5.1c).

The teachers developed knowledge of learners' learning difficulties through analysis of their classwork, homework and assignments, as well as through post-teaching discussions on statistical graphs construction (ref Sections 5.2.1-5.2.4). Constant examination of the learners' workbooks helped to reinforce the teachers' insight into learners' conceptions (preconceptions and misconceptions) of statistics topics (ref Sections 4.5.1–4.5.4).

The teachers addressed these difficulties at various times through extra classes, problem-solving tasks using familiar real-life examples, post-teaching discussions, and teaching on a one-to-one basis after normal school hours (ref Section 5.3.4). The process of identifying and addressing learners' learning difficulties should have provided the teachers with ample knowledge of learners' preconception and learning difficulties in statistics teaching. But it is surprising that after so many years of teaching mathematics, some of the teachers are not aware of these problems. This lack of familiarity with learners' anticipated learning difficulties could be because the topic was recently introduced into the curriculum. Therefore, the teachers may not have developed the required PCK for addressing the difficulties which learners' may experience in learning school statistics. However, by identifying and addressing learners' alternative conceptions and learning difficulties, the participating teachers may have gained more knowledge of the learners' learning difficulties in statistics teaching.

6.4` Concluding remarks

Based on the findings of this study, individual teachers constructed their PCK in statistics teaching by:

- Formally developing their knowledge of the subject matter in an accredited formal education programme in which they had the opportunity to study the subject matter and methodology of school statistics
- Teaching school statistics using procedural and conceptual knowledge to some extent (ref Sections 4.5.1-4.5.4.).

- Using several mathematics and statistics textbooks, past senior certificate examination question papers in statistics and other materials in lesson preparation, consistent with their understanding of the nature of statistics in school mathematics and how it should be taught (ref Section 5.2.1-5.2.4). For example, Teacher A taught his lesson of histogram construction and assigned classwork and homework using learners' mathematics textbook (ref Section 4.5.1, first lesson observation, and line 23b).

- Using varied topic-specific instructional skills such as construction skills (involving the drawing of axes, choosing of scale, labelling of axes, plotting thee points and joining the line of best fit), problem-solving, assessment (in the form of oral probing questioning, classwork, homework and assignments), and interpretation skill (comprising of determining the relationship between X and Y, and based on the relationship between X and Y values, one can say whether there is positive correlation, negative correlation, or no correlation as in, second lesson observation, and line 4bii), in teaching scatter plots (ref Section 4.5.3)

- Using diagnostic techniques (oral questioning, pre-activity and class discussions) and a review of previous lessons to introduce lessons, and to determine learners' preconceptions in statistics teaching (ref Section 4.7.3)

- Using a variety of assessment techniques such as classwork, homework and assignments and grouped work in statistical graphs to assess how well learners understood the lesson on statistical graphs and to identify their difficulties (ref Sections 4.7.3).

- Continually updating their knowledge of school statistics by attending content knowledge workshops and other teacher development programmes designed to improve content awareness and practice (ref Section 5.3)

By knowing how teachers develop PCK for teaching school statistics, teacher educators will be able to develop greater understanding and insight into designing programmes to teach topics that were previously included only at tertiary level.

6.5 Educational implications of the study

Based on the results of this study, the educational implications can be summarised as follows:

The findings of this study can be used to provide a knowledge base and process to be employed by mathematics teachers to develop PCK for the continuous improvement of effective mathematics classroom practice. For instance, the teachers developed knowledge of learners' learning difficulties by analysing their responses to classwork, homework and assignments and during pre- and post-activity discussions. Regular examinations of learners' workbooks helped to reinforce the teachers' familiarity with learners' conceptions and learning difficulties of statistics topics. Learning difficulties were generally addressed by the teacher engaging the learners on a one-to-one basis or collectively during or after school hours.

The development of subject matter content knowledge of statistics renders it an essential component of PCK for teaching it at school level. When teaching statistics, teachers' actions were determined to a large extent by the depth of their PCK, thereby making subject matter content knowledge an essential component of their ongoing learning of school statistics for the improvement of their expertise in statistics and effective classroom practice.

'Pedagogical content knowledge research links knowledge of teaching with knowledge of learning' (Adela, 2009). This is a powerful base on which to build teaching expertise. In this study, formal education in mathematics was found to be a prerequisite in developing teachers' subject matter content and pedagogical knowledge. Several research reports have attempted to establish how PCK is developed in science and mathematics. As PCK is topic-specific, however, little attempt has been made to determine how PCK is developed in the context of teaching statistics by mathematics teachers. The research that is available suggests that this type of information is meagre. This study has therefore furnished insight into how PCK is developed by competent mathematics teachers. A detailed description was given of examples of the PCK of mathematics teachers in terms of improving learners' performance in statistics and for consideration by teacher trainers in designing statistics teacher education programmes for in-service and pre-service teachers.

In this study as indicate in section 1.6, PCK was conceptualised to include content specific knowledge, content specific instructional strategies and learners' preconceptions of specific concept, rules and skills. 'PCK development is a complex process and it is not clear how it is developed in statistics teaching for mathematics classroom practices (Jong, 2003). PCK is distinct from a general knowledge of pedagogy, educational purpose and learners' characteristics. Moreover, because PCK is concerned with the teaching of a particular topic e.g statistics, it may turn out to differ considerably from the subject matter itself' (Jong, Van Driel and Verloop, 2005:948). PCK is said to develop by an iterative process that is rooted in classroom practice. The implication is that many beginning teachers have little or no PCK at their disposal, particularly in statistics teaching (ref Section 1.6).

From the description of the knowledge-base and process employed by competent mathematics teachers in developing PCK in statistics teaching, notions of and insight into PCK could be obtained that can be incorporated into a mathematics education programme for in-service and pre-services mathematics teachers, thereby contributing to the continuous improvement of the mathematics teacher education programme and teachers' PCK.

6.6 Suggestions for further study

The results of this study present several areas for further research opportunities. These areas are suggested:

- Large-scale research needs to be conducted on the kind of subject matter content knowledge that a teacher needs for development of PCK in statistics, especially in the construction and interpretation of graphs of grouped data, which many teachers seem to find difficult to teach.

- More studies need to be conducted to determine the impact of teachers' knowledge of learners' preconceptions as a theoretical framework for investigating teachers PCK in statistics teaching.

- This study found that procedural and conceptual knowledge were both necessary for teaching statistical graphs, especially in addressing learners' misconceptions and learning difficulties. Further studies are needed to determine how well both approaches can be applied to other aspects of school statistics.
- More researches need to be conducted on why teachers with over five years

experience of teaching mathematics lack sufficient knowledge of learners' preconceptions in statistics teaching.

6.7 Limitations of the study

This study may have been influenced by these limitations, which should be taken into consideration when interpreting the results:

- Selection of the participants created a problem that led to having only a few in the study. The number of participants was reduced because of the criteria used in selection. The schools from which the participants were selected had to obtained a pass rate of 70% and above in mathematics in the senior certificate examination for at least two years. This left the researcher with a small number of schools from which to select willing participants.
- Assessment of teaching competencies is usually associated with inherent limitations as they are coloured by personal observer idiosyncratic tendencies. The results of this study during the lesson observations may not necessarily be replicated. The process of interpreting teachers' practice and decisions, and placing them into specific pedagogical categories may not always be 100% correct. The possible errors in the interpretations were reduced by the triangulation of data, using open assessments (questionnaires to confirm the teacher observations and the categories assigned) and negotiations for placing pedagogical actions into appropriate categories of how the teachers developed their PCK in the teaching of statistics. Discourse on classifying pedagogical actions into appropriate categories depended on the negotiations that took place between the researcher and the teachers, and was bound to differ from one teacher to another. The interpretations of the lessons and post-teaching discussions could be viewed as temporal (dependent on time and pairs) and tentative. The possible significant errors could be minimised by using multiple strategies to collect data.
- Another limitation included external validity or the ability to generalise the results. Only four teachers participated in the entire study. The number of cases was limited to making broad generalisations. Not only the number of cases, but also the geographical location and the school types may be too limited to produce a general theory on PCK appropriate for teaching statistics in school mathematics. The number of participants

also provided the possibility of variation or similarity in PCK assessment for mathematics teachers using the same working document such as the mathematics work schedule, the results in the senior certificate examination in mathematics, recommendations from principals, subject specialists and peers.

- Organising lessons outside normal school hours posed its own challenges. Learners were sometimes tired at the end of the school day. Extra-curricular activities at the schools occasionally affected the teaching programme. Therefore adjustments had to be made to assure consistency and uniformity in all the statistics topics.

6.8 The role of the researcher in the non-participatory lesson observation.

In this study, non-participatory classroom observation in statistics lessons was conducted with the four participating teachers. As explained in paragraph 2 of section 6.7, assessment of teaching competencies involving a non-participating observer is usually associated with inherent limitations, owing to the presence of the observer. The teacher and the students might behave differently from the ways in which they would normally comport themselves. (Cresswell, 2008; University and College Union, 2012). In this study there was always the possibility that the participating teachers teaching could have been somewhat influenced, as indicated later, by the presence of the researcher and the research interest. The interest was with determining how mathematics teachers considered competent developed their PCK in statistics teaching by observing them in statistics lessons among other things. Any one of such influences could possibly occur in the planning and presentation of statistics topic lessons. For example, to perhaps try to impress the observer they could select instructional materials and use instructional strategies they think are effective but not necessarily economical that they would not normally use routinely in teaching the assigned statistics lessons.

The presence of the researcher could also have influenced learners' responses or active participation (freely or inhibited) during the lessons. While these are the possibilities that could have arisen during the lesson, I believe that I tried to minimise those instances by first introducing myself to the participating teachers and their learners during negotiations with them on the extent and nature of the lessons to be taught and observed (Creswell, 2008, University and College Union, 2012) and spending some time with the teachers (familiarisation) before embarking on any formal classroom observation. Additionally, during

the meetings at which the teachers were briefed about the objectives of the research, they were assured that the observation was not an assessment in any form or shape of their teaching performance, but was designed to gain better understanding of how to help teachers with a new topic, statistics, that has recently been introduced into the mathematics curriculum. The learners were also encouraged by both the researcher and their teachers to feel free and less anxious to participate, just as in a normal lesson, since no assessment was involved. The participating teachers were given access to all the recorded field notes, the video recordings and their transcriptions to comment on, and to approve, before the analysis of data. Furthermore, triangulation of data helped to minimise and/or address any inconsistencies in the participating teachers' questionnaire and interview responses, and classroom behaviour.

6.9 Summary of the chapter

In this chapter, the summary, conclusion and recommendations for further investigations were presented. The results of the study indicate that mathematics teachers may have constructed PCK in teaching statistics through the acquisition of formal subject matter knowledge of the topic in formal education programmes, and they develop their subject matter content knowledge during classroom practice. The teachers taking part in the study possessed the necessary content knowledge, and demonstrated it through procedural and conceptual approaches to teaching statistical graphs, although the rule-oriented procedural approach was dominant in teaching data-handling topics. Mathematics and statistics-related textbooks and other learning materials were other sources used by the teachers to acquire the subject matter content knowledge that was needed to plan and deliver their lessons.

Knowledge of instructional strategies, notably the use of a formal rule-oriented approach and instructional skills such as the construction skills, was developed through formal education and years of experience in classroom practice. Analyses of learners' classwork, homework and assignments were used mostly to gain teacher knowledge of learner misconceptions and topic-specific learning difficulties. Intervention strategies such as the used extra tutoring, class activities in the form of drill and practice, repeating and re-explaining of lessons in which learners are experiencing difficulties as well as post teaching discussions were used to addressed the alternative conceptions and learning difficulties. The chapter concluded with

highlights of the educational implications, suggestions and limitations of the study for future researchers to note.