CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the study

This study focused on how competent secondary school mathematics teachers develop pedagogical content knowledge (PCK) for teaching statistics in high school. While some researchers perceive statistics as a subject on its own (Moore & Cobb, 2001; Gordon, Petocz & Reid, 2007), others believe it should be taught as part of the mathematics curriculum and consequently view it as a mathematical concept (Franklin, Kader, Mewborn, Moreno, Peck, Perry & Schaeffer, 2005; Gattuso, 2006).

According to the National Curriculum Statements (NCS) of South Africa (DoE, 2009), the country in which this study was conducted, statistics is taught as part of the mathematics curriculum under the rubric of 'data handling'. In accordance with the new curriculum, the learning outcomes of mathematics require that learners should be able to use appropriate measures of central tendency and spread to collect, organise, analyse, and interpret data in order to establish statistical and probability models for solving related problems (DoE, 2007).

According to the NCS, instructional guides and other publications, teachers need to be given in-service professional support by mathematics experts or professionals with the statistics knowledge required to implement the new mathematics curriculum. This is because the topic of statistics has been included in the national curriculum for the first time, and it is assumed that most teachers will not have the requisite knowledge for teaching it. Thompson (2005) indicated that in order to implement the new curriculum effectively, teachers need subject matter knowledge, pedagogical knowledge, and pedagogical content knowledge (PCK).

Subject matter knowledge refers to the disciplinary knowledge obtained through formal training in colleges and universities, while pedagogical knowledge pertains to the knowledge of instruction and learning that the teacher needs in order to deal with everyday classroom educational tasks (Vistro-Yu, 2003). Such tasks involve the use of various teaching styles and strategies and the management of learning processes in the classroom (Vistro-Yu, 2003). These skills and competencies are normally acquired through formal training and teaching practice. Simply described, PCK is about the overall knowledge the educator has of the subject matter content that learners should master in a particular topic or subject, and how it

should be taught, so that effective and efficient learning can take place (Mitchell & Mueller, 2006). In short, PCK is an amalgam of subject matter content and pedagogy, which is uniquely the province of teachers and involves their own special form of professional understanding for good teaching (Jong, 2003).

PCK is specific to teaching and differentiates between expert teachers in a particular subject area and subject area experts (Griffin, Dodds & Rovengno, 1996). To illustrate, mathematics teachers differ from mathematicians, not necessarily in the quantity and quality of their subject matter knowledge, but more specifically in how that knowledge is organised and used (Cochram, De Ruiter & King, 1993). An experienced mathematics teacher's knowledge of the subject is organised from a teaching perspective and is used as a basis for helping learners to understand specific concepts. A mathematician's knowledge, on the other hand, is normally organised from a research perspective and is used mainly as a basis for developing new knowledge in the field. This implies that PCK may be something beginner or inexperienced teachers may not necessarily learn only from textbooks or from short courses. From the literature reviewed, little is known as to how PCK is developed, or even facilitated, in the context of teaching statistics (Godino, Batenero, Roa & Wilhelmi, 2011; DoE 2008; Jong, 2003). Therefore, further research is needed in order to identify and define the skills and practices necessary for PCK development in statistics education more carefully (DoE, 2008).

To develop PCK, Jong (2003) argues that teachers need to explore instructional strategies for specific topics and their learners in practice. Various studies – such as those by Dooren, Verschattel and Oghenna (2005), Boerst (2003), Halim and Meerah (2002) and Van Driel, Verloop and De Vos (1998) have shown that inadequate PCK is one of the areas that require most attention in teacher education, as many teachers are unable to enhance learner performance because of lack of subject matter content knowledge and PCK. Many beginner teachers, including inexperienced mathematics teachers, do not know how to develop and use PCK in their teaching (Van Driel et al., 1998; Halim & Meerah, 2002). In consequence, they become uncomfortable with teaching certain topics, and, for that reason, may omit them altogether (ICM/IASE, 2007).

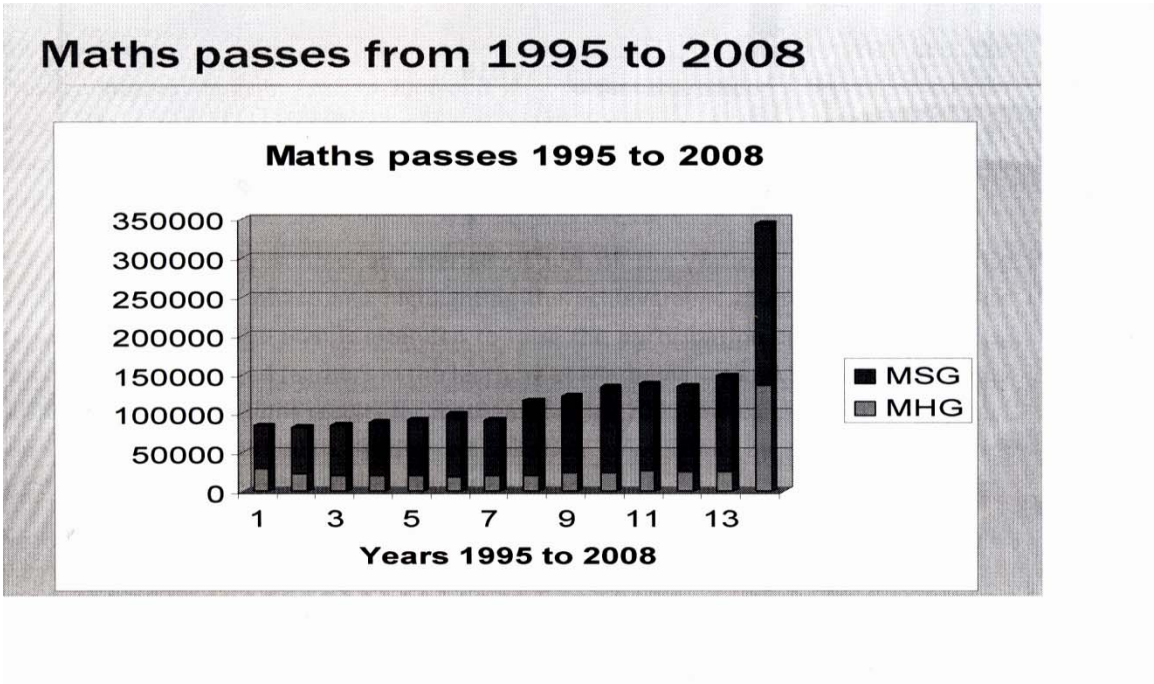
Data on mathematics enrolment and learner performance over a period of five years in the South African Senior Certificate (SC) examination, as displayed in Table 1.1 and Figure 1.1

below, show that learners generally underachieve in mathematics. Mathematics failure rates in the SC examination remain unacceptably high, and the number or percentage of learners that leave Grade 12 with a higher-grade pass in mathematics is unacceptably low. While the percentage of candidates that wrote the mathematics examination over the period of six years increased, the percentage of learners that passed mathematics for standard grade (MSG) was below 30%, and below 10% in mathematics for higher grade (MHG) (Figure 1.1). This suggests a crisis of mathematics underachievement at secondary school level.

Table 1.1: Learners' performance in mathematics from 1999 to 2004 in the South African Senior Certificate Examination

	1999		2000		2001		2002		2004	
No. of candidates	511,225		489,941		449,371		442,590		467,985	
% of learners that wrote mathematics	55%		58%		59%		59%		81%	
% of learners that passed mathematics	SG	HG	SG	HG	SG	HG	SG	HG	SG	HG
	20%	4%	21%	5%	24%	4%	27%	5%	29%	5%

Source: DoE (2006); CDE (2007)



NB: This is the period in which the standard and higher grades examinations are used to assess mathematics learners in the Senior Certificate Examination.

In addition, the chief examiner's report on learners' performance in both mathematics and mathematical literacy in the 2008 and 2009 SC examinations shows that learners generally underperform in statistics (DoE, 2009). According to this report, there was a steady increase in learners' enrolment and performance in mathematics, compared with previous years, but about 60% of those who passed scored between 30% and 40% (the pass mark for mathematics, according to the NCS, is 30%) (DoE, 2009). Furthermore, the learners' performance in questions relating to statistics in paper 2 was below 35%. As a result of the poor performance in statistics, teachers' ability to teach this topic, the quality of senior certificate products, and university enrolment in mathematics and statistics-related subjects have been subject to review (Keeton, 2009).

Many studies, such as those by Howie (2002) and DoBE (2012), on the causes of poor performance in mathematics in South Africa, show that one of the main factors that is attributable to learners' poor performance is the teacher. Others include language and classroom environment (CDE, 2004). The interest in this study is with the teacher factor. The study is aimed at investigating specifically how competent teachers develop and use PCK to improve the quality of instruction and learning in statistics. The competent mathematics teachers were identified from their learners' final results in mathematics in the public senior certificate exam and on recommendations by principals, peer teachers and subject experts in the Department of Education. Although being competent may not necessarily mean that they are expert in statistics, their selection as competent teachers depends on their final Senior Certificate Examination results in mathematics over time. The research seeks to determine what it is that these teachers who have been classified as competent teachers have and do when using their PCK to teach particular subject matter content in statistics. The assumption here is that PCK can be measured. PCK has been used as a theoretical framework for this study.

The topic of statistics has been chosen because it is completely new in the mathematics curriculum, and many teachers may not have adequate experience in teaching it, let alone in handling the difficulties learners experience with it. Until the introduction of the topic of data handling in mathematics and mathematical literacy in 2006, statistics was not taught in high schools (DoE, 2006). Many, if not all, teachers of mathematics would not have formal knowledge of statistics, let alone knowledge of learners' preconceptions, which need to be

addressed in teaching and learning statistics. The assumption is that few in-service teachers would have developed the PCK needed to teach the topic effectively. Therefore, it would be useful to study how teachers who are considered competent go about teaching a new topic in statistics, and to document what it is that they have and do as they go about preparing their lessons, and how they teach those data-handling lessons.

In 2007, this lack of familiarity with statistics content and teaching on the part of secondary school mathematics teachers worldwide was given added support by papers presented at the joint conference of the International Commission for Mathematics Instruction and the International Association for Statistics Educators (ICMI/IASE, 2007). The conference highlighted that mathematics teachers are likely to face challenges in terms of teaching a topic such as statistics in which they do not necessarily have an understanding of learners' learning difficulties, and may not know how to present the content in a way that learners can understand.

Recent studies, such as those by Jong (2003), Jong, Van Driel and Verloop (2005), Capraro, Capraro, Parker, Kulm and Raulerson (2005), Wu (2005), and Godino et al. (2011), showed that most mathematics teachers at high-school level have limited PCK. A clear understanding of how teachers develop PCK and use it to enhance learner achievement in mathematics is useful knowledge for any pre-service and in-service teacher education programme. This study is an attempt to provide a comprehensive description and analysis of how the mathematics teachers selected for the study developed their PCK in teaching statistics.

1.2 The research problem

The NCS Curriculum for Mathematics was introduced in Grade 10 in all high schools in the Republic of South Africa in 2006. Mathematics teachers were charged with the responsibility of delivering the curriculum in the classroom in line with the NCS recommendations and ensuring effective teaching, so that learner achievement could be enhanced (DoE, 2006). However, since the introduction of this curriculum, learners have not been performing as they should, because of internal and external classroom factors that result in underachievement (Howie, 2002; CDE, 2004; DoE, 2008).

Reddy (2006) identifies PCK as one of the limiting factors in enhancing learner achievement

in mathematics in the South African context. Other researchers elsewhere in the world, such as Wu (2005), Capraro et al. (2005), Halim and Meerah (2002), and Van Driel et al. (1998), have come to the same conclusion, especially with regard to statistics (Cazorla, 2006), which has only recently been included in the curriculum as a formal aspect of mathematics. The lack of familiarity with statistics has placed teachers' confidence in their ability to teach it in doubt (ICMI/IASE, 2007). Poor learner performance in statistics was also noted at the joint conference of the ICMI and the IASE (ICMI/IASE, 2007), at which conference delegates attributed learners' poor performance to the rudimentary state of mathematics teachers' PCK in statistics. In addition, the chief examiner's report for the Senior Certificate Examination in Mathematics shows that learners underperform in statistics (DoE, 2008). The report suggests that poor PCK background may have contributed to learners' underperformance in statistics, and that this background therefore needs to be investigated (DoE, 2010).

Given the instructional demands of the new mathematics curriculum and the poor performance of learners in statistics, this study was concerned with investigating how competent mathematics teachers at high-school level in South Africa develop PCK in statistics teaching in order to enhance learners' achievement in mathematics.

1.3 Aims of the study

The aims of the study were:

a) To determine how competent secondary school mathematics teachers develop PCK for teaching statistics
b) To determine the implications that PCK has for mathematics education programmes

1.4 Statement of the problem

The problem identified for this study was to determine how secondary school mathematics teachers who are assumed to be competent develop the PCK they use in teaching statistics in school mathematics. In addition, the implications of these findings for mathematics teacher education programmes were determined and discussed.

1.5 Research questions

The problem statement gave rise to the following research questions:

1) What subject matter content knowledge of statistics do mathematics teachers who are considered competent have and demonstrate during classroom practice?
2) What instructional skills and strategies do these teachers use in teaching statistics?
3) What knowledge of learners' preconceptions and learning difficulties, if any, do these teachers have and demonstrate during classroom practice?
(4) How do these teachers develop PCK in statistics teaching?

1.6 Significance of the study

The significance of this study is that it is hoped its findings will provide a knowledge base and process employed by mathematics teachers to develop pedagogical content knowledge in statistics teaching for the improvement of learners' performance; and ideas and knowledge that can be incorporated into a mathematics education programme for in-service and pre-service mathematics teachers.

Besides, PCK development is a complex process and it is not clear how it is developed in statistics teaching for mathematics classroom practices. 'PCK is distinct from a general knowledge of pedagogy, educational purpose and learners' characteristics' (Jong, Van Driel & Verloop, 2005: 948). 'Moreover, because PCK is concerned with the teaching of a particular topic for example statistics, it may turn out to differ considerably from the subject matter itself' (Jong, Van Driel & Verloop, 2005: 948). PCK is said to develop by an iterative process that is rooted in classroom practice (Miller, 2006). The implication is that beginning teachers have little or no PCK at their disposal, particularly if they are new to statistics teaching. A clear understanding of how PCK is developed in statistics teaching will be a requisite for designing effective statistics education programme for in-service and pre-service statistics educators.

A great deal of research has been conducted in an attempt to identify and characterise PCK during classroom practice, but research communities continue to call for studies to devise methods of measuring PCK (Miller, 2006). According to Miller (2006), PCK represents much more than a category of teacher knowledge; it provides a starting point for research

involving teacher education. As a theoretical framework of this study, PCK provides a process for organising teacher education research.

1.7 Theoretical framework

Several researchers (Shulman, 1986; Van Driel, 1998; Jong, 2003; Abell, 2008; Hill, 2008; Watson, Callingham & Donne, 2008; and Toerien, 2011) have made serious attempts to develop models to measure teachers' PCK in mathematics and the sciences. These researchers have largely been challenged by the difficulties the models present in distinguishing the boundaries that make up the various constructs (Graham, 2011). These difficulties include the changeable nature of PCK, which makes it difficult to pinpoint specific constructs of this category of teacher knowledge (Miller, 2006). In addition, because of the numerous categories of knowledge that could be integrated into PCK, differences may exist in the boundaries of a PCK construct (Hill *et al.*, 2008); and indeed because teachers, like learners, construct their own knowledge, there is every likelihood that there will be individual examples of teacher PCK. It is precisely because of these constraints that research on PCK development has not always been as straightforward as researchers might have hoped. A review of the literature indicates that the use of PCK in research and for methods of data collection and analysis has mostly taken two forms (Shulman, 1986; Van Driel, 1998; Penso, 2002; Jong, 2003; Cazorla, 2006; Abell, 2008; Hill, 2008; Watson, Callingham & Donne, 2008; and Toerien, 2011). The first form has to do with research on PCK as a category of teachers' knowledge, that is, knowledge specifically constructed by teachers and yet distinctly different for each subject matter content area. The second form involves research using PCK as a theoretical framework, which is based on a number of assumptions, as we shall see later. The fundamental difference between these two forms of using PCK in research is that while the first entails trying to identify or measure PCK, the second utilises the assumption that PCK exists, in order to examine other aspects of teacher knowledge (Miller, 2006). In this study, the interest was in first determining teacher PCK in the context of teaching school statistics, which is assumed to exist, and second in determining the way in which it (PCK) is developed and used in teaching school statistics topics. To this end, the study used PCK as a theoretical framework, consisting of teacher subject matter content knowledge, pedagogical knowledge, and knowledge of learners' conceptions and learning difficulties to explore the main research questions based on a number of assumptions.

The initial model of PCK, which was supported by several studies (eg Shulman 1987), tagged PCK as the specific teacher knowledge that allowed a teacher to more thoroughly understand how to transform content knowledge into a more conceptually accessible version for students or learners. As explained by Shulman (1987) PCK results from the blending of content knowledge and pedagogical methods. Thus, it is a widely accepted belief that PCK represents the category of knowledge that is needed for a novice teacher to mature into an expert (Bodner & Orgill, 2007). Shulman's (1987) vision and Ball *et al.'s* (2008) description of teacher knowledge as an amalgam of categories of knowledge, including content, curricular, pedagogical, and student knowledge, and PCK, has virtually compelled many teacher education programmes to create new instructional activities for improving classroom practice. This same vision of enriching classroom practice has provided a focus on education research. Unfortunately, PCK, because of its nebulous nature, remains a category of knowledge that is difficult to isolate and research (Miller, 2006). Nevertheless, it provides a starting point for researchers who wish to collect and analyse data on other aspects of teacher knowledge. In this study the teachers' classroom practice in statistics was therefore investigated in a series of lesson observations, in order to explore what PCK exists and how the participating teachers demonstrated their PCK in the context of teaching statistics in school mathematics. The first consideration was that identifying the category(ies) of knowledge that the teacher has, as defined, in the teaching of statistics would yield information about teacher's PCK and how it is developed and used during classroom practice.

It was mentioned earlier that the use of PCK as a theoretical framework has provided researchers with a new perspective for collecting and analysing data about teacher knowledge or cognition (Jong, 2003; Rollnick *et al.*, 2008; Toerien, 2011). The use of PCK as a theoretical framework allows researchers to focus on specific questions about a teacher's knowledge base and is founded on a series of assumptions. Miller (2006) has indicated that PCK embodies an epistemological approach to understanding teacher knowledge. Precisely for this reason, in this study, the teachers' PCK in statistics teaching, and the way in which they developed it, was conceptualised as comprising content knowledge, pedagogical knowledge, and knowledge of learners' preconceptions and learning difficulties in the context of teaching school statistics. These central categories of teacher knowledge were used as the theoretical framework that provided a guide for data collection, analysis and discussion of what and how PCK in statistics teaching was developed.

Assumptions of the study

Based on the above considerations, in this study the use of PCK as a theoretical framework was built on the following assumptions, as summarised (Miller 2006).

- PCK represents a category of teacher knowledge that is the essence of an expert teacher in a specific topic (Miller, 2006), in this case in school statistics teaching. In this study, the blending of subject matter content knowledge, pedagogical knowledge, and knowledge of learners' preconceptions and learning difficulties was used to describe the PCK of the participating teachers.

- PCK provides a framework that can be used to describe the origin of this critical teachers' knowledge (Miller, 2006). In other words, PCK represents an epistemological approach, to constructing teaching knowledge.

- PCK is a constructivist process and therefore a continually changing body of knowledge. Teachers, like learners, construct their own knowledge and in this study it is assumed that the development of PCK is a continuously modifying unit, beginning with teacher preparation programmes, evolving through teaching experience and assimilating and accommodating professional development opportunities.
    - Identifying and measuring PCK constructs can be achieved by using instruments designed for that purpose. In this study, the components of PCK were assessed using multiple assessment strategies, which include concept mapping, teacher interviews, teacher questionnaires, lesson observation, written classroom activity reports and document analysis.

According to Shulman (1986), PCK is a specific category of knowledge that goes beyond the knowledge of subject matter per se to include the dimension of subject matter knowledge for teaching. It refers to teachers' interpretations of subject matter in the context of facilitating learning. In consequence, it has been argued that PCK is one of the seven categories in Shulman's (1986) categorisation of a knowledge base for teaching. The key elements of

Shulman's conception of PCK are:

i) Knowledge of the representation of the subject matter for teaching
ii) Knowledge of relevant instructional strategies
iii) Knowledge of learners' conceptions (preconceptions and misconceptions)
iv) Knowledge of learners' learning difficulties

For the purpose of this study, these four elements appear to be most appropriate in defining the PCK that may be used for teaching statistics in school mathematics, namely subject matter content knowledge; knowledge of teaching (pedagogical knowledge); knowledge of learners' conceptions (preconceptions and misconceptions); and knowledge of learners' learning difficulties. These four elements cover the views and constructs of PCK used by various researchers in this domain, such as Jong (2003), Shulman (1986), Jong et al. (2005), Halim and Meerah (2002), Rollnick et al. (2008), Hill (2008) and Toerien (2011).

For the construct of PCK, the working definition is that PCK is an amalgam of subject matter content knowledge, pedagogical knowledge (instructional skills and strategies), knowledge of learners' conceptions and knowledge of learners' learning difficulties. In this study, the researcher's intention was to determine the PCK that competent teachers use in teaching statistics by observing the PCK that such teachers demonstrate in the classroom. It is assumed that because such teachers are considered competent and have experience in teaching mathematics, they will be able or will be likely to integrate content knowledge and pedagogical knowledge in ways that contribute to the development of the PCK used for teaching statistics (Jong, 2003). To this end, the development of PCK was inferred from the teacher interviews, questionnaires, written reports, document analysis and lesson observation.

1.7.1 Subject matter content knowledge

According to Manouchehri (1976), subject matter content knowledge consists of an explanatory framework and the rules of evidence within a discipline. The subject matter content knowledge of prospective mathematics teachers is acquired primarily during disciplinary education (Jong, 2003). This knowledge consists of substantive content knowledge and syntactic content knowledge (Barnes, 2007). Substantive content knowledge refers 'to the concepts, principles, laws, and models in a particular content area of a

discipline'. Syntactic content knowledge, by contrast, is the 'set of ways in which truth or falsehood, validity or invalidity are established' (Schwab, 1978, cited in Shulman, 1986). In practice, teachers should be able not only to define the acceptable truths for learners in a domain, but to explain, in theory and in practice, why these truths are worth knowing and how they relate to other propositions, within the discipline and outside it.

Both types of subject matter knowledge (substantive and syntactic) are needed for teachers' development of PCK, because they help to create an adequate understanding of the nature of the subject matter and beliefs about how it should be taught (Jong, 2003). It is therefore assumed that mathematics teachers with good PCK have both types of subject matter content knowledge and are able to apply this knowledge in making the topic understandable to learners. This assumption is given empirical support by Wu (2005), who indicated that teachers with good PCK have a firm command of subject matter knowledge and are able to design mathematics instructional material that allows learners to grasp what they teach. Muijs and Reynolds (2000) referred to these teachers as effective teachers.

Other scholars, such as Carpenter, Fennema, Petterson and Carey (1988), Even (1993), Manouchehri (1997), Van Driel et al. (1998), Halim and Meerah (2002), Tsangaridou (2002), Viri (2003) and Hill (2008), have studied the influence of subject matter knowledge on the PCK of pre-service, novice and expert teachers. These studies revealed that teachers' content knowledge goes a long way towards determining the level of teachers' PCK. The subject matter content knowledge is one of the components of PCK that will be assessed in this study.

1.7.2 Pedagogical knowledge

Cochram et al. (1993) define pedagogical knowledge as knowledge about teaching. Vistro-Yu (2003) defines it as the knowledge used for teaching, particularly expertise in teaching techniques, psychological principles, classroom management, and teaching and learning processes. Following these definitions, pedagogical knowledge is believed to be the kind of information that a teacher needs and uses to perform everyday teaching tasks, involving teaching styles and strategies, classroom management and teaching and learning processes relating to learners in the classroom. Research findings by Rollnick et al. (2008), Jong et al. (2005) and Vistro-Yu (2003) show that a mathematics teacher with adequate pedagogical knowledge is able to design good teaching and learning strategies and manage the classroom

and other instruction and learning processes. This constituent framework seems appropriate for defining the construct of pedagogical knowledge, as it describes in operational terms what the teacher needs to do to create an environment that is conducive to learning. In this study, the focus was on the instructional skills and strategies used for teaching statistics in school mathematics.

To this end, the pedagogical knowledge of mathematics teachers was assessed by examining their lesson planning and implementation, questionnaires, written reports, and interviews, in order to probe the way in which competent teachers develop their pedagogical knowledge and use it in the instruction and learning process.

1.7.3 Knowledge of learners' conceptions in statistics teaching in school mathematics

Learners' conceptions in statistics in school mathematics consist of preconceptions and misconceptions. A mathematical misconception is a belief or idea that is based on incorrect or erroneous information about a given mathematical concept (Olivier, 1989). According to Olivier (1989), most mathematical misconceptions arise because of pre-existing concepts or preconceptions in the mind of the learner or the teacher. Misconceptions can occur when an attempt is made to link preconceptions and new knowledge to be learned. Olivier (1989) argues that misconceptions play a key role in understanding a new concept. The role of the mathematics teacher in resolving mathematical misconceptions is usually to develop some form of teaching and learning approach, such as teacher-learner or learner-learner discussion, communication, reflection, and negotiation of meaning, that addresses the missing concept (Penso, 2002; Cazorla, 2006). Through these approaches, the mathematics teacher may be able to get to the root of the misconception.

Cazorla (2006) for example reported that misconceptions and the way in which mathematics lessons are taught are among the factors that cause learning difficulties. According to her, most statistics teachers do not have adequate knowledge of the school curriculum and the approaches needed to teach and learn statistics, which can result in poor content delivery in the classroom situation. Jong (2003) noted that in order to identify and resolve misconceptions and learners' learning difficulties during classroom practice, the teacher could use convergent and inferential techniques. Convergent and inferential techniques are data-collection systems that entail developing questions for a topic in short-answer and

multiple-choice formats to probe the preconceptions and misconceptions of learners (Jong, 2003). The teachers' written reports and the learners' notebooks may help to identify where the learners' learning difficulties lie (Jong, 2003; Jong et al., 2005, Penso, 2002; Van Driel et al., 1998).

The participating teachers in this study will be examined to determine whether they have prior knowledge of statistics as they teach the assigned topic through lesson planning and implementation.

1.7.4 Knowledge of learners' learning difficulties

Penso (2002) reports that learning difficulties may stem from the way lessons are taught. For example, learning difficulties may arise from the content of the lesson, lesson preparation and implementation and the learning atmosphere (Penso, 2002). Other factors include misconceptions that learners and teachers have about a topic, as well as cognitive and affective characteristics of learners. According to Penso (2002), 'learners consider their learning difficulties to be the result of conditions that existed prior to the process of teaching, as well as those existing in the course of teaching'.

In this study, the ways in which the teachers identified and addressed the learning difficulties that learners encountered during classroom practice were determined in lesson observation.

From the above discussion, subject matter content knowledge, pedagogical knowledge (instructional skills and strategies), learners' conceptions (preconceptions and misconceptions) and learners' learning difficulties were used to conceptualise the construct of PCK for teaching school statistics. These frameworks were derived from the model proposed by researchers such as Shulman (1986), Van Driel (1998), Jong (2003), Cazorla (2006), Penso (2002), Abell (2008), Hill (2008) and Toerien (2011), as discussed in sections 1.7.1□1.7.4. The selection of these components of PCK was based on the assumption that PCK is dynamic, topic specific, and transformative, and can be measured using these frameworks (Corrigan, 2008). While the subject matter content of the participating teachers was measured using a conceptual knowledge exercise, concept mapping, interviews and lesson observation, instructional skills and strategies were assessed using lesson observation, questionnaires, interviews, written reports and document analysis. Lesson observation, questionnaires, written reports and reviews of teachers and learners' portfolios, as well as

lesson plans and learners' workbooks, were used to assess the teachers' knowledge of learners' preconceptions and learning difficulties in statistics teaching. The roles of each instrument in measuring the individual component are described in Section 3.5.1.

To summarise, subject matter content knowledge, pedagogical knowledge, and learners' conceptions and learning difficulties were used to conceptualise the PCK needed for teaching statistics in school mathematics.

1.8 Scope of the study

This study explored how selected mathematics teachers at high-school level develop PCK in statistics teaching. The focus was on teachers who were teaching mathematics in accordance with the NCS Curriculum (now called Curriculum and Assessment policy Statement (CAPS)) for Mathematics at high schools in Tshwane North Education District in South Africa. These teachers were selected as participants for this study based on the performance of their learners in the public Senior Certificate Examination and on being recommended as competent teachers by principals, peers, and mathematics specialists at the Department of Basic Education (DoBE). Since PCK is topic specific (Corrigan, 2008), data were collected during statistics lessons by means of lesson observation. The participants in this study were few, because of the criterion used, namely a pass rate of 70%, and because participation was voluntary.

1.9 Criteria for selecting the topic

The concept of statistics is defined by Otumudia (2006) as the science of collecting, organising, and analysing data for any given purpose. Statistics helps us to reduce large and scattered data to an understandable level, thereby enabling us to make decisions in the face of uncertainty (Otumudia, 2006).

Statistics is taught as part of the mathematics curriculum under the rubric of 'data handling'. Data handling is one of the four major topics in the Curriculum and Assessment Policy Statements (CAPS) (DoBE, 2011). The reasons for including data handling in the new curriculum are:

i) Basic statistical knowledge is necessary for all kinds of data interpretation, as people

encounter a great deal of categorical and numerical observations that should be used to guide decisions (DoE, 2006).

ii) Data handling helps one to build critical thinking, to understand reality, and to be able to participate in social actions.

iii) Statistics and probability are useful in daily life and play an instrumental role in other disciplines, such as economics, engineering, and medicine (Franklin & Mewborn, 2006).

iv) There is a need for basic stochastic knowledge in many professions, and statistics plays an important role in developing critical thinking that help in the development of this type of knowledge (Innabi, 2002).

For these reasons, the learning outcomes require learners studying statistics to be able to collect, organise, analyse, and interpret data to establish statistical and probability models for solving related problems (DoBE, 2011).

1.10 Definition of terms

In this section, some of the terms that are used to describe how mathematics teachers develop PCK for statistics teaching are defined operationally.

- **National Curriculum Statements (NCS)**

The National Curriculum Statements (NCS) are guidelines that state what each learner should achieve in terms of learning outcomes and assessment standards by the end of each grade. In this study, the NCS for Mathematics is used to describe the curriculum for mathematics as the subject that is taught in Grades 10 to 12.

- **Curriculum and Assessment Policy Statement (CAPS)**

The National Curriculum and Assessment Policy Statement is a 'single, comprehensive, and concise policy document, which replaced the Subject and Learning Area Statements, Learning Programme Guidelines and Subject Assessment Guidelines for all the subjects listed in the National Curriculum Statement Grades R – 12' (DoBE, 2012).

- **Pedagogical content knowledge (PCK)**

The construct of PCK constitutes an amalgam of subject matter content knowledge, pedagogical knowledge (instructional skills and strategies), knowledge of learners' conceptions and knowledge of learners' learning difficulties. In this study, PCK is used to describe and measure the way mathematics teachers combine subject matter content knowledge and pedagogical knowledge, as well as use their knowledge of learners' preconceptions and learning difficulties to carry out effective teaching during classroom practice.

- **Pedagogical knowledge**

Pedagogical knowledge is that knowledge that a teacher needs and uses to perform everyday teaching tasks, involving instructional skills and strategies, and classroom management and teaching and learning processes relating to learners in the classroom (Vistro-Yu, 2003). Pedagogical knowledge is used to define the construct of PCK in statistics teaching in this study. Specifically, the instructional skills and strategies will be used to describe the pedagogical knowledge in statistics teaching in this study.

- **Conceptions in the teaching and learning of statistics**

Conceptions in teaching and learning statistics consist of preconceptions and misconceptions. A preconception is regarded as the prior knowledge of a given topic with which learners come to the class (Olivier, 1989) and is used as such in this study. It is manifested during lesson observation. A misconception can occur as a result of a pre-existing concept. Both preconceptions and misconceptions can contribute to learners' learning difficulties in classroom practice. The term 'misconception' was used to describe the learners' beliefs or notions that were based on incorrect or erroneous information about a given statistical concept demonstrated during classroom practice. Teachers' knowledge of learners' conceptions in learning statistics was used to describe the PCK that was likely to be used for teaching statistics in school mathematics.

- **Competent mathematics teachers**

In this study, competent mathematics teachers were identified based on their learners' final results in mathematics in the public senior certificate exam and recommendations made by

principals, peer teachers and subject experts at the Department of Education. Although being competent may not necessarily mean that the teachers are knowledgeable or expert in statistics, they were able to help their learners to do well in their final Senior Certificate Examination in Mathematics. The teachers were observed while teaching school statistics in order to determine how they develop their PCK.

- **Conceptual knowledge**

  Conceptual knowledge involves an understanding of mathematical ideas and concepts, as well as the interrelationships among these concepts. It consists of the ability to identify and apply principles, facts and definitions, and to compare and contrast related concepts (Engelbrecht & Potgieter, 2005). In this study the conceptual knowledge approach involves the use by the teacher of mathematical ideas, principles, facts and definitions to explain mathematical concepts and their relationships during the teaching and learning of a particular topic.

- **Procedural knowledge**

  Procedural knowledge is a formal symbolic representation system of a given mathematical task using algorithms, or rules, to complete the mathematical tasks (Star, 2002). In practice, it means for the teacher the use of particular rules, algorithms or procedures to complete a given task without necessarily providing an explanation underpinning the rules or procedures used. For example, the construction of statistical such as bar graph, histogram, ogive and scatter diagrams requires that one should first draw the axes, choose the scale, label the axes, plot the points and join the line of best fit (Leinhardt *et al*, 1990). The four participating teachers followed this procedure during their lessons on bar graphs, histograms, ogives and scatter diagrams. This teaching approach essentially uses what is referred to in this study as a **procedural knowledge approach.**

- **Document analysis**

Document analysis is a technique used in this study to gather information. It describes the act of reviewing the documentation of comparable school systems in order to extract pieces of information that are relevant to the current research project. Hence it is sometimes regarded as a research project requirement. In this study, document analysis was used to extract information about teaching and learning of statistics from the NCS for Mathematics, teacher and learner portfolios, and learners' class workbooks.

1.11 The chapter structure of the thesis

The study is divided into six chapters. Chapter 1 presents the introduction and background of the study and the way in which the background relates to the problem under investigation. The context in which the study took place is described.

PCK, as one of the forms of knowledge needed to implement the curriculum, was defined from four perspectives, namely content-specific knowledge; content-specific instructional strategies; knowledge of learners' conceptions of statistics teaching and learning; and knowledge of learners' learning difficulties. The chapter then presents the guiding research questions and theoretical framework based on the purpose of the study and the statement of the problem. The key concepts in this study are highlighted and discussed. The chapter concludes with a brief discussion of the structure of the thesis.

Chapter 2 focuses on the literature review, which captures the empirical and theoretical aspects related to the process of PCK development and how it is used in classroom practice to teach mathematics and science. The literature review derives its focus from the National Curriculum Statement for Mathematics, theoretical framework and the research questions, which seek to describe the way in which competent mathematics teachers develop PCK in statistics teaching. Chapter 2 is divided into two sections. The first section discusses literature about the content of statistics according to NCS and research on the teaching of statistics in school mathematics. The second section discusses the models of capturing PCK, conceptualisation of PCK and techniques for measuring PCK.

Chapter 3 discusses the methodology of the study. It is argued that a rich description of data comes from using several strategies of investigation, data collection, and data analysis. The chapter describes the methodological plans for the study, the pilot study, the participants, the research activities, and the various instruments used in the collection of data. The validity and reliability of the instruments are also discussed in this chapter.

Chapter 4 presents the results of the data collection discussed in chapter 3. The first presentation concerns the quantitatively analysed data, and the second concerns the qualitatively analysed data. The latter relies on the quantitative data that has been analysed. While the quantitative data are derived from the conceptual knowledge exercises, the concept mapping exercises, and the results of the schools from which the participants were selected,

the qualitative data are derived from the interviews, lesson observations, free-response questionnaires, teachers' written reports and documents related to teachers' guides, and learners' portfolios, mathematics workbooks, and textbooks. In this chapter, the guiding research questions and the theoretical framework are revisited in order to determine how competent mathematics teachers develop their PCK in statistics teaching.

Chapter 5 contains a discussion of the results, based on the results of the previous chapter. The guiding research questions are revisited. In line with the theoretical framework, the chapter presents a discussion focusing on the teachers' PCK profiles and how the data help to answer the research questions in order to determine how the mathematics teachers developed their pedagogical content knowledge in statistics teaching.

Chapter 6 presents a summary, the conclusions of the study, and recommendations and suggestions for further research.

1.12 Summary of chapter

This chapter provided insight into the research orientation used in this study, in an attempt to make the reader conversant with the research project. The chapter began with an introduction to the NCS and the learning outcomes for statistics in school mathematics. The knowledge that the teacher needs to implement the curriculum effectively was highlighted from three perspectives, namely subject matter content knowledge, pedagogical knowledge, and PCK. The introduction was followed by an elucidation of the problem of the study, a statement of the research problem, the aims of the research, the research questions, the significance of the study, the scope of the study, and the theoretical framework that guided the study. The key concepts used in this study were then defined and discussed, and the chapter concluded with a discussion of the criteria for selecting the topic, as well as the chapter structure of the thesis.