CHAPTER 5

A LEARNING MODEL OF PRIVATE CONSUMPTION EXPENDITURE IN SOUTH AFRICA

Most economists accept that beliefs about the future are an important determinant of behaviour today. (Begg et al. 1991:568)

5.1 INTRODUCTION

In this chapter, the hypothesis that South African consumers are forward-looking with respect to prices when making consumption expenditure decisions, is tested. It is assumed that consumers learn using a Kalman filter-based (boundedly rational learning) process for updating their expectations conditional on prior errors made when forecasting the price level.

The first stage of implementing the boundedly rational learning approach would involve the estimation of the time-varying mechanism, which represents economic agents using incomplete historical information to form expectations. In the second stage, the expectations formation mechanism is incorporated into the behavioural equations. The theoretical specification of the private consumption expenditure function (or categories of consumption expenditure) would therefore include a price expectations variable, namely the expected one-period-ahead consumer price level.

Two sets of empirical results will be presented in this chapter: first, the time-varying coefficients of the price expectations rule – that is the state equations of the state-space form of the price expectations rule – and second, the set of behavioural equations containing the price expectations variable. Consumption expenditure is disaggregated into the following categories: durable consumption (including durables and semi-durables), non-durable consumption and services. Empirical estimation results for total private consumption expenditure are presented, followed by durable consumption expenditure and non-durable consumption expenditure. Since stochastic estimation of total consumption expenditure is believed to be more reliable than that of the consumption expenditure on services, expenditure on services is deterministically determined as the residual of the total and the other two categories.

Behavioural equations are subjected to extensive diagnostic testing to ensure that the model is statistically well specified. Deterministic analysis of the response characteristics of the model is also conducted to ensure that short and long-run response characteristics correspond to theoretical priors and long-run equilibrium properties of the data.

5.2 THE THEORETICAL MODEL

The theoretical model for private consumption expenditure is developed in this section. Its empirical estimation results are presented in section 5.5. *A priori* expectations of elasticities of variables included in the long-run equilibrium relationships of the different categories of consumption expenditure will be pointed out, as well as expected short-run dynamic properties of the equations.

Since price expectations feature as a variable contributing towards the short-run dynamic structure of the behavioural equations, a theoretical model for the formation of price expectations by the South African consumer will also be proposed.

5.2.1 The consumption function

The theoretical specification of the behavioural equations for each of the categories of consumption, based on the forward-looking theories of consumption, will include an income variable, some wealth variable, and possibly long-term or short-term interest rates representing monetary conditions. These are according to theory (as discussed in Chapter 2), the variables to consider for the long-run equilibrium or steady-state relationship:

$$c_t = Ay_t^{\alpha} w_t^{\beta} r_t^{\gamma} e^{\varepsilon_t} \tag{5.1}$$

$0<\alpha<1; \ 0<\beta<1; \ \alpha+\beta=1; \ \gamma<0$

with

c_t	=	private consumption expenditure in period t
y_t	=	personal disposable income in period t
w_t	=	financial wealth stock in period t
r_t	=	a representative interest rate in period t and
ε_t	=	the stochastic disturbance term.

The explicit inclusion of both the income and the wealth variables is justified by the forward-looking theories of consumption, and further motivated by the Ball-Drake hypothesis regarding the derivation of utility from the accumulation of wealth (section 2.5.3). Both α and β are expected to be positive with $\alpha+\beta=1$.

Within an intertemporal consumption optimisation framework[6], the interest rate constitutes a trade-off between current and future consumption. A rise in the rate of return on accumulated savings increases the opportunity cost associated with current consumption and should raise the savings rate, thus lowering current consumption. On the other hand, the future income stream expected from the higher rate of return on savings may encourage current consumption (equations (2.8) and (2.9) of the permanent income hypothesis). Interest rates may therefore have a negative or a positive effect on consumption expenditure. The substitution effect is however expected to be larger than the income effect; hence a negative expected sign on interest rates.

5.2.1.1 *A priori* expectations of income elasticity and wealth elasticity

As discussed in section 1.4, South Africa is characterised by absolute poverty and an unequal distribution of both income and wealth. A substantial portion of the population possesses virtually no wealth and earns a small income, if any. More specifically, 90 per cent of the population earns less than 50 per cent of total income (Whiteford and van Seventer 1999:ii), while the unemployment rate ranges in the region of 36 per cent (Stats SA 1998). The large portion of the population constrained by very low income levels,

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[6] The first order condition characterising optimal consumption behaviour would be $u'(c_t)/E[u'(c_{t+1})] = (1+\rho)/(1+r_t)$ with ρ the rate of time preference.

spends virtually all of its income on consumption, with very little left to be utilised for wealth accumulation. Wealth therefore plays an insignificant role in their consumption expenditure decisions. For South Africa as a whole, one would expect an income elasticity close to unity for *total* consumption expenditure, and a low wealth elasticity, while for *durable* consumption, a relatively larger wealth elasticity would be expected. For *non-durable* consumption, wealth would not be expected to be a driving factor in the long run. Wealth may however contribute towards the short-run dynamic structure of non-durable consumption expenditure.

5.2.1.2 Short-run dynamics

Other variables that were considered as explanatory of consumer behaviour include lagged consumption expenditure, i.e. expenditure patterns of the past, variables reflecting labour market sentiment, for example the employment rate, relative prices, lagged personal savings, credit leasing finance, instalment sale credit, stock market prices, etc. The expected price level, that is the one-period-ahead consumer price level forecasted by the Kalman filter, can be used to test for the effect of price expectations on consumption expenditure behaviour within a boundedly rational learning framework. Most of the above variables proved to contribute towards explaining the short-run dynamics of the system.

A priori, one would expect interest rates only to be significant in explaining durable consumption expenditure and, perhaps, total consumption expenditure. Conversely, interest rates are not expected to have a significant influence on non-durable expenditure decisions, given that the majority of South African consumers are subjected to liquidity constraints and have limited access to bank or any other form of credit as a consequence of their extremely low income levels. In addition, these consumers spend virtually their entire income on non-durables for immediate consumption at a subsistence level, ruling out a substitution effect following interest rate changes. These consumers have no savings; an interest rate increase therefore fails to affect their future income, and thus an income effect is equally improbable.

Likewise, price expectations are expected to play a significant role in explaining durable and aggregate consumption expenditure. As motivated previously, the majority of South

African consumers are trapped in a relatively rigid pattern of consumption. Liquidity constraints disqualify them from increasing current consumption, to for example hedge against expected price increases. Absolute poverty rules out further reductions in current consumption; these consumers are frequently consuming at a subsistence level. As before, substitution and income effects are effectively disabled. Price expectations are, therefore, unlikely to have a significant effect on these consumers who allocate virtually all income towards non-durable expenditure.

Variables reflecting labour market conditions, like the employment or unemployment rate, may also play a role in explaining consumption, particularly non-durable consumption. Adverse developments in the labour market often affect the unskilled workforce first. Their wages are likely to be low and mainly directed towards non-durable consumption. Fluctuations in labour market conditions will therefore be reflected by changes in non-durable expenditure patterns.

5.2.2 A model for price expectations formation

A boundedly rational learning approach towards the formation of price expectations is intuitively attractive. It is also consistent with psychology literature on learning processes, unlike the rational expectations hypothesis, which demands full information from economic agents regarding the model as well as its parameters.

The boundedly rational learning approach is based on the assumption that expectations are formed by intelligent agents who are not fully informed, but learn about their environment as time progresses. The learning model requires the specification of a time-varying parameter rule of expectations formation. In many applications, the expectations rule is derived from the reduced form of the structural equation. An application to the exchange rate sector of the London Business School model as discussed in section 3.4, serves as an example in this regard. Citing Hall and Garratt (1992b:11), "How one derives an expectations equation is by no means exclusive. In principle, formation of expectations could be the result of information anywhere in the model, if it is thought to be relevant".

In setting up the expectations rule in this case, the application of price expectations to wage behaviour of countries in the global econometric model (GEM) (Barrel *et al.* 1994:174) is followed. The criteria used to decide on the inclusion of variables in the time-varying expectations rule, according to Barrel, do not follow from a tightly formulated theory, but inclusion of variables are also not completely *ad hoc*. Rather, variables are selected to capture important endogenous linkages in the model and also enable the price expectations equation to adjust the coefficients on existing variables in accordance with movements in variables not included in the behavioural equation.

The expectations rule may therefore be represented by the following theoretical specification:

$$\Delta cpi_{t+1} = e^{\xi_{1t}} \Delta cpi_{t-1}^{\xi_{2t}} \Delta rl_{t-1}^{\xi_{3t}} \Delta exch_{t-1}^{\xi_{4t}} e^{w_t} . \tag{5.2}$$

The dependent variable for the price expectations equation is the change in the consumer price index one period ahead (Δcpi_{t+1}). The information set or the independent variables include the change in the representative long-term interest rate (Δrl_{t-1}) and the change in the rand/US dollar spot nominal exchange rate ($\Delta exch_{t-1}$). The above specification implies that information of period t−1 is utilised to form price expectations in period t with respect to period t+1.

Alternative specifications that deserve consideration and empirical testing include the 3-month bankers' acceptance rate, or the prime interest rate or even the M3 money supply, instead of the representative long-term interest rate. The spot rand/US dollar exchange rate may possibly be replaced in the specification by e.g. import prices, the terms of trade, or the effective exchange rate. In addition, the wage rate and capacity utilisation in the economy may also be considered as possible explanations of the expected price level. The GEM specification (*op. cit.*:174) mentioned earlier, does include a capacity utilisation variable in the information set, in addition to prices, interest rates and the relevant exchange rate.

The above theoretical specification in a South African context may be motivated as follows. The specification, including lagged prices, interest rates and the exchange rate is an attempt to model the psychological expectations formation process of the (often unsophisticated) consumer. Given that 19 per cent of the adult South African population is illiterate (has not completed primary school) (Stats SA 1998), the adjustment of parameters of an expectations rule based on variables like the terms of trade, capacity utilisation or the money supply probably implies an unrealistically sophisticated consumer. Information about changes in prices, interest rate levels and the exchange rate is perhaps more accessible to the average consumer than any other economic variables influencing price changes. Price expectations are expected *a priori* to be autoregressive for the most part, with a less significant contribution by other variables included in the information set.

The first step in implementing the learning model of price expectations would be to formulate the expectations rule in state-space form.

Again, consider the general form of the state-space representation for a model with stochastically varying coefficients:

$$y_t = a(x_t) + [H(x_t)]'\xi_t + w_t \tag{5.3}$$

$$\xi_{t+1} = F(x_t)\xi_t + v_{t+1} \tag{5.4}$$

The price expectations rule is a regression in which the coefficient vector changes over time:

$$y_t = x'_t\,\beta_t + w_t \tag{5.5}$$

$$(\beta_{t+1} - \bar{\beta}) = F(\beta_t - \bar{\beta}) + v_{t+1} \tag{5.6}$$

where x_t is a $(k\times 1)$ vector that includes lagged values of y or variables that are independent of the regression disturbance w_τ for all τ.

If the eigenvalues of the $(k\times k)$ matrix F are all inside the unit circle, then $\bar{\beta}$ is interpreted as the average or steady state value of the coefficient vector. If it is further assumed that

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} \mid x_t, \vartheta_{t-1} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & \sigma^2 \end{bmatrix}\right), \tag{5.7}$$

then (5.5) to (5.7) will be recognised as a state-space model of the general form of (5.3) to (5.4) with state vector $\xi_t = (\beta_t - \bar{\beta})$. The regression in (5.5) can be written as

$$y_t = x'_t \overline{\beta} + x'_t \xi_t + w_t \tag{5.8}$$

which is an observation equation of the form (5.3) with $a(x_t) = x'_t \overline{\beta}$, $H(x_t)=x_t$ and $R(x_t)=\sigma^2$.

In terms of the specification of equation (5.2), y_t and x_t in equations (5.3) will be given by:

$$y_t = \Delta cpi_{t+1}$$

$$x'_t = [1, \Delta cpi_{t-1}, \Delta rl_{t-1}, \Delta exch_{t-1}]'. \tag{5.9}$$

The unknown parameters of the system will be estimated along with the (4×1) state vector, ξ_t. The state vector will be assumed to evolve through time according to a random walk with drift process, that is $\xi_{t+1}=\xi_t+v_{t+1}$.

Since all variables are integrated of order 1 (see section 5.3.2), first differences are taken and all variables are utilised in natural logarithm form. The empirical result of the expectations rule will be reported in section 5.4.

5.3 THE DATA

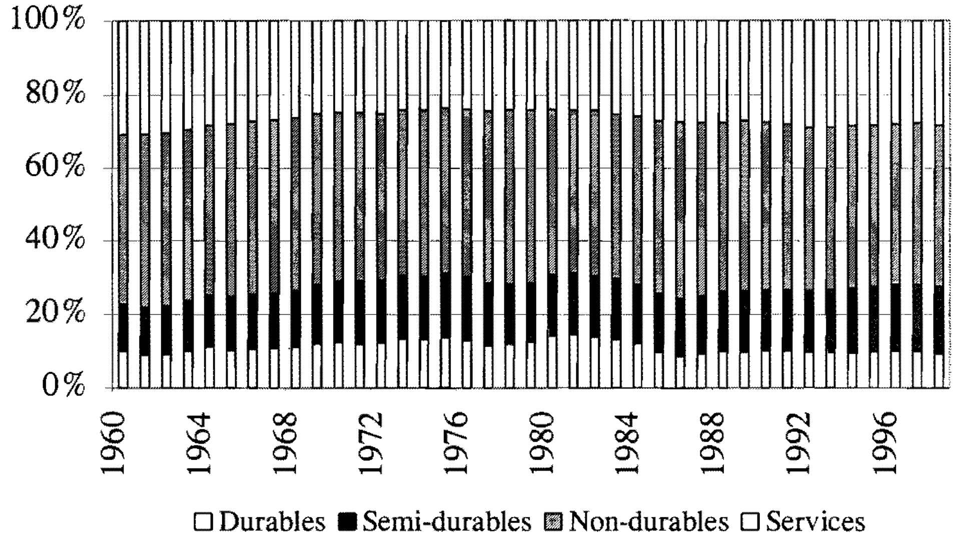
The sources and construction of the data series used to empirically estimate the theoretical models above are discussed in this section, as well as the univariate characteristics of the data.

5.3.1 Sources of data and calculations

Total private consumption expenditure is disaggregated into 4 categories, namely durable consumption, semi-durable consumption, non-durable consumption and services. Expenditure on durables on average accounts for 11 per cent of total consumption expenditure, while semi-durable expenditure accounts for 16 per cent of the total. This average percentage for non-durables is 46 per cent and for services 27 per cent. All data is published in real terms. Figure 5.1 gives an indication of this distribution over time.

The income variable is represented by real disposable income. The consumer price index (CPI) is used to deflate this variable. The 3-month bankers' acceptance rate and the eskom rate are considered as the representative short-term and the long-term interest rates respectively.

Figure 5.1 Private consumption expenditure



Source: South African Reserve Bank, Quarterly Bulletin, Various issues.

Non-human wealth would ideally include net financial wealth of households, housing wealth and possibly an index for stock market prices. The magnitude of the *stock* of wealth in South Africa is however not available in time series format. An indication of the *flow* of financial wealth for the household sector is available from the National Financial Account in the form of the financing balance (i.e. financial assets - financial liabilities)[7].

The (financial) wealth stock variable was then constructed by accumulating financial flows for the household sector from 1970, assuming a base value in that year. The return on wealth variable was constructed by means of the representative long-term interest rate. A

[7] Financial assets and liabilities as reported in the National Financial Account consist of 24 items including gold and other foreign reserves; cash and demand monetary deposits; short, medium and long-term monetary deposits; deposits with other institutions and other financial institutions; treasury and other bills; bank loans and advances; trade credit and short-term loans; short and long-term government stock; non-marketable government bonds; securities of local authorities and public enterprises; other loan stock and preference shares; ordinary shares; foreign branch/head office balances; long-term and mortgage loans; interest in retirement and life funds; amounts receivable/payable; other assets/liabilities and a balancing item.

constraining factor is that data on financial wealth is only available from 1970 onwards, whereas the other variables under consideration date back to 1960 and some to 1946. Wealth variables were also deflated using the CPI.

The source of all the data used is the *Quarterly Bulletin* of the South African Reserve Bank. For a list of variables refer to Table 5.1. All variables are used in natural logarithmic form.

Table 5.1 List of variables

Series	**Description**
$ctot_t$	Real total private consumption expenditure
$cdur_t$	Real private consumption expenditure on durables and semi-durables
$cndur_t$	Real private consumption expenditure on non-durables
$cserv_t$	Real private consumption expenditure on services
yd_t	Real personal disposable income
w_t	Real financial wealth
rw_t	Real return on financial wealth
$empl\_na_t$	Employment in the non-agricultural sectors
$relp_t$	Prices of durables relative to prices of non-durables, $(Pdur/Pndur)_t$
rs_t	Nominal short-term interest rate (3-month bankers' acceptance rate)
rl_t	Nominal long-term interest rate (eskom rate)
$exch_t$	Rand/US dollar spot nominal exchange rate
cpi_t	Price level (consumer price index)
cpi_e_t	Expected one period ahead price level ($cpi\_e_t = cpi_{t+1}$)
$cpi\_e\_f_t$	Kalman filter prediction of expected one period ahead price level

5.3.2 Order of integration

In analysing the univariate characteristics of the data, the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests were employed to establish the order of integration of the data series. The testing strategy discussed in section 4.3.2 was used; namely as suggested by Dolado *et al.* (1990) and as applied by Sturm and De Haan (1995:69).

The number of lags used in the estimated equations was determined in a similar way as suggested by Perron (1989:1384), namely starting with eight lags and testing downwards, until the last lag is significant or there are no lags left.

In addition, graphing the data series in levels as well as their first and second differences, looking at autocorrelation functions (correlograms) and spectrum analysis, proved to be helpful when ADF-test results were inconclusive.

Tables 5.2, 5.3 and 5.4 report the outcomes of the ADF-tests for all relevant data series employed in estimations. The series tested are listed in the first column. The second column reports whether a trend and a constant (Trend), only a constant (Constant), or neither one (None) is included. In the third column, the number of lags used is reported. The next column shows the ADF t-statistic, called τ_τ when a trend and a constant are included, τ_μ when only a constant is included, and τ when neither is included. The last column reports the F-statistic, Φ_3 (Φ_1), testing whether the trend (constant) is significant under the null hypothesis of no unit root.

According to Table 5.2, ADF-tests rendered two of the variables stationary in levels, namely the log of non-durable consumption and the log of the consumer price index. However, by simply looking at graphical representations of these series it becomes obvious that these series cannot be stationary in levels. Table 5.3 indicates that all variables are indeed integrated of order 1. From experience and evidence from other tools, prices are known to be integrated of either order 1 or 2. The second differenced form of the consumer price index was therefore also subjected to a unit root test. In Table 5.4, it is once again clear that the ADF test is inconclusive in establishing the order of integration of the consumer price index. The increasing rate of the change in the general price level between 1970 and 1986 and the slowdown in the rate of change from then onwards, could be the reason for the ADF-test's failure to be conclusive in this instance, see Figure 5.7. In this study, the consumer price index is regarded as I(1) and used in first differenced form. All variables that were employed in empirical estimation are presented in Figure 5.2.

Figure 5.2 Graphical representation of data series employed in estimations, all data in natural logarithmic form

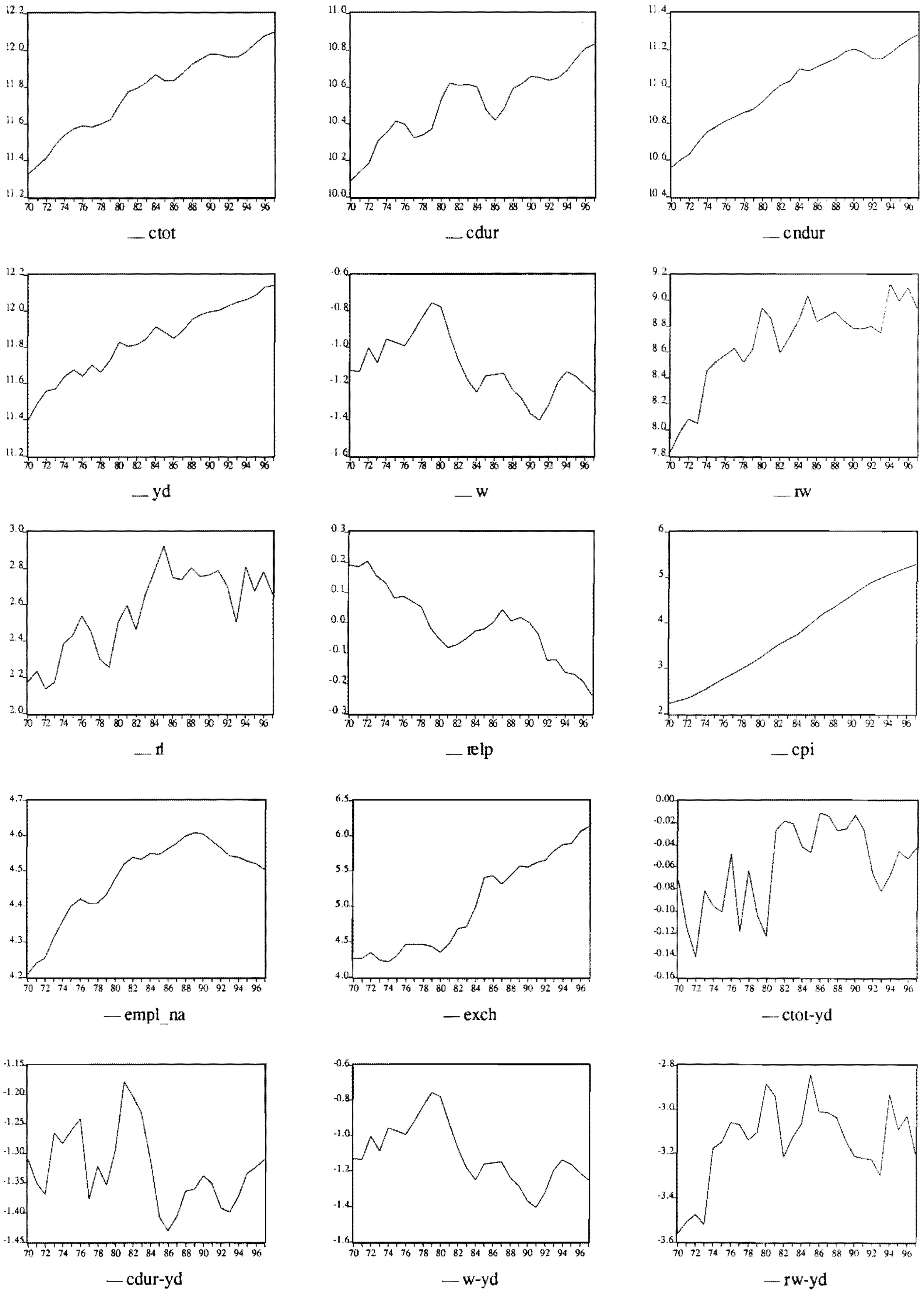


Table 5.2 Augmented Dickey-Fuller tests for non-stationarity, levels, 1970-1997 (All data series in natural logarithmic form)

Series	Model	Lags	$\tau_\tau, \tau_\mu, \tau$[a]	Φ_3, Φ_1[b]
$ctot_t$	Trend	4	-1.94	4.17
	Constant	2	-2.19	4.99
	None	6	1.00	
dur_t	Trend	1	-3.39	7.23
	Constant	4	-2.45	4.07
	None	2	1.74	
$cndur_t$	Trend	0	-1.62	4.64
	Constant	0	-2.90**	8.46**
	None	1	2.48	
yd_t	Trend	2	-3.59	5.01
	Constant	2	-2.71	2.83
	None	0	3.88	
w_t	Trend	8	-3.23	5.19
	Constant	8	-2.87	4.23
	None	5	0.38	
rw_t	Trend	0	-2.83	5.02
	Constant	8	-2.45	1.74
	None	0	1.93	
rl_t	Trend	8	0.14	1.59
	Constant	8	-2.15	1.78
	None	8	1.34	
$relp_t$ $(Pdur/Pndur)_t$	Trend	6	-3.35	1.96
	Constant	8	-0.42	1.05
	None	2	-1.16	
$empl\_na_t$	Trend	3	0.81	10.93**
	Constant	8	0.07	4.22
	None	5	-0.77	
$exch_t$	Trend	3	-2.34	3.02
	Constant	2	0.73	1.43
	None	0	3.13	
cpi_t	Trend	1	-1.52	17.26**
	Constant	4	-2.46**	12.09**
	None	6	1.02	

*(**) Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), -3.00(-3.77) when only a constant is included (τ_μ) and -1.96(-2.68) when neither is included (τ). The standard normal critical value is -1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

Table 5.3 Augmented Dickey-Fuller tests for non-stationarity, first differenced, 1970-1997 (All data series in natural logarithmic form)

Series	Model	Lags	$\tau_\tau, \tau_\mu, \tau$[a]	Φ_3, Φ_1[b]
$\Delta ctot_t$	Trend	3	-4.21*	4.68
	Constant			
	None			
Δdur_t	Trend	1	-3.93*	5.26
	Constant			
	None			
$\Delta cndur_t$	Trend	3	-3.88*	4.80
	Constant			
	None			
Δyd_t	Trend	0	-5.83*	16.98**
	Constant			
	None			
Δw_t	Trend	3	-3.95*	4.09
	Constant			
	None			
Δrw_t	Trend	0	-6.20**	19.27**
	Constant			
	None			
Δrl_t	Trend	2	5.03**	13.02**
	Constant			
	None			
$\Delta relp_t$	Trend	0	-4.31**	9.31*
$\Delta(Pdur/Pndur)_t$	Constant			
	None			
$\Delta empl\_na_t$	Trend	2	5.02**	6.49
	Constant			
	None			
$\Delta exch_t$	Trend	2	-4.61**	7.81*
	Constant			
	None			
Δcpi_t	Trend	0	-2.25	4.76
	Constant	0	-2.89**	8.38**
	None			

*(**) Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), -3.00(-3.77) when only a constant is included (τ_μ) and -1.96(-2.68) when neither is included (τ). The standard normal critical value is -1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

Table 5.4 Augmented Dickey-Fuller tests for non-stationarity, second differenced, 1970-1997 (Data series in natural logarithmic form)

Series	Model	Lags	$\tau_\tau, \tau_\mu, \tau$[a]	Φ_3, Φ_1[b]
$\Delta\Delta cpi_t$	Trend	2	-0.59	2.18
	Constant	0	-2.09	4.36
	None	0	-0.35	

*(**) Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), -3.00(-3.77) when only a constant is included (τ_μ) and -1.96(-2.68) when neither is included (τ). The standard normal critical value is -1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

5.4 ESTIMATION TECHNIQUE USED

The estimation technique employed for estimation of behavioural equations is the Johansen maximum likelihood estimation methodology. The main advantage of the Johansen technique over the Engle and Yoo three-step procedure has been pointed out in Chapter 4 and is, in essence, the inability of the latter to determine the number of cointegrating relationships present in the data. (If there are n variables included in the model, as many as (n−1) linearly independent cointegration vectors may possibly exist.) Assuming that there is one cointegrating vector present, when there are in fact more, leads to inefficiency in the sense that only a linear combination of these vectors may be obtained when estimating a single equation model. Other advantages of the Johansen procedure have also been discussed in Chapter 4.

Although the Johansen technique as a means of establishing the number of cointegrating relationships is readily available in econometrics software packages, the technique imposes several demands. The distinct steps in the estimation process have also been highlighted in Chapter 4, and these will be followed in the practical estimation. The process starts by confirming the order of integration of all variables. Also, the correct lag length of the vector autoregressive (VAR) model is determined so as to ensure that the vector error correction model has Gaussian errors. The Johansen cointegration test is then used to test for the reduced rank of the system, i.e. to determine the number of cointegrating relationships present in the data. Next, the inclusion of deterministic variables (constant and trend) must be established by considering whether the data contain trends. This is followed by testing for weak exogeneity and by testing the linear hypotheses on the cointegrating relationships. The final steps in the process normally entail testing for unique cointegrating vectors and imposing joint tests of restrictions on the α loading matrix and the β cointegrating vector.

In this study, the residual obtained from the cointegrating vector, - i.e. the equilibrium error estimated from the long-run equilibrium relationship, is implemented in an unrestricted single equation error correction model (ECM) and the residual diagnostics are used to test whether error terms are white noise. The coefficient of the equilibrium residual in the ECM also provides information about the speed of adjustment towards equilibrium.

The estimation results obtained for the three estimated private consumption expenditure categories (i.e. total, durables and non-durables) are presented in section 5.5.2.

5.5 ESTIMATION RESULTS

The application of the Kalman filter estimation to the state-space representation of the time-varying price expectations rule is presented in this section, including the subsequent implementation of the Kalman filter price expectations forecast in the behavioural equations.

5.5.1 Time-varying parameter estimation of the expectations rule

The first step in the estimation process would be to estimate the structural equation $\Delta cpi_t = f(\Delta cpi_{t-2}, \Delta rl_{t-2}, \Delta exch_{t-2})$ with fixed parameters by means of ordinary least squares (OLS), including all variables in natural logarithmic form. Table 5.5 reports the expectations equation estimated by OLS. This is intended to give an indication of the approximate form of price expectations when estimated with time varying parameters. It appears from Table 5.5 that price expectations are mainly autoregressive. The most significant term is the lagged consumer price index while the other variables are, for the most part, insignificant. They are retained, however, because it is believed that they may serve to capture some endogenous interaction in the broad macro model.

Table 5.5 OLS regression of price expectations equation

Dependent variable: Δcpi_t

Variables	**Coefficient**	**Std. Error**	**t-Statistic**
constant	0.023696	0.010126	2.340120
Δcpi_{t-2}	0.767885	0.101243	7.584574
Δrl_{t-2}	0.018168	0.036861	0.492869
$\Delta exch_{t-2}$	-0.000037	0.000238	-0.154866
sample period (adjusted): 1963 to 1998 $\overline{R}^2 = 0.6216$ s.e. = 0.0274			

Table 5.6 reports the hyperparameters and residual diagnostics for the price expectations equation when estimated using the Kalman filter. No convergence was found over the sample period 1960 to 1998 or 1965 to 1998, and therefore the estimation was conducted over the sample period 1970 to 1998. Convergence was in this instance achieved after 70 iterations with a convergence factor of 0.001. Finding no convergence is not uncommon when conducting estimation with the Kalman filter (Hamilton 1994:387-388). The EViews user's guide (1997:544) suggests that different starting values for ξ_0 and P_0 or a different solution algorithm[8] be used when problems of this nature are encountered. By default,

[8] In this instance, the default algorithm used by EViews for models which may be estimated using first derivative methods was selected, namely the Marquardt algorithm. This algorithm modifies the Gauss-Newton algorithm by adding a correction matrix (or ridge factor) to the Hessian approximation. The ridge

Eviews estimates initial values by running OLS on the observations equation, treating ξ_0 as a fixed parameter.

The variances of the error term of the observation equation and the covariance matrix of the state equation error terms, which is assumed to be diagonal, are reported in the top part of the table. $R(1,1)=\sigma^2$ represents the variance of the error term of the observation equation and Q represent the covariance matrix of the error terms or adjustment factors of the state equations. The latter, which are often called hyperparameters, determine the speed of learning and reflect the signal to noise ratio for each variable; hence they also reflect the rate of convergence of the model. The hyperparameters only make sense when compared to the variance of the error term of the observation equation, which in this case is for all practical purposes equal to zero. In this comparison, the magnitude of the hyperparameters is indicative of a fairly rapid learning process with respect to lagged prices, and to a lesser extent with respect to interest rates and the exchange rate.

Put differently, the covariance matrix of the state equation error terms represents the 'forgetting factor' (Hall and Garratt 1992b:7). When the matrix contains only zeros, the Kalman filter will generate OLS estimations of the state vector, ξ. As the diagonal elements of the covariance matrix increase, the parameters are allowed to change more rapidly, and in effect 'forget' the past.

The final values of the state vector, ξ, with associated standard errors, are also reported in the top part of the table. ξ_1, ξ_2, and ξ_3 represent the time-varying coefficients of the independent variables in the expectations rule, Δcpi_{t-2}, Δrl_{t-2} and $\Delta exch_{t-2}$ respectively. The intercept coefficient of the price expectations rule is defined as time-independent (constant).

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correction handles numerical problems when the outer product is near singular and may improve the convergence rate.

As an alternative, the BHHH-algorithm (Berndt, Hall, Hall and Hausman) may be selected. This algorithm is referred to as Gauss-Newton for general nonlinear least squares problems, and Berndt, Hall, Hall and Hausman (BHHH) for maximum likelihood problems. It follows Newton-Raphson, but replaces the negative of the Hessian by an approximation formed from the sum of the outer product of the gradient vectors for each observation's contribution to the objective function. For least squares and log-likelihood functions, this approximation is asymptotically equivalent to the actual Hessian when evaluated at the parameter values which maximise the function. When evaluated away from the maximum, this approximation may be quite poor (EViews 1998:621).

Table 5.6 Hyperparameters, final value of the state vector and equation diagnostics from the time varying Kalman filter estimates

Dependent variable: Δcpi_t

Variables	Coefficient	Std.Error	t-Statistic
R(1,1)	6.90E-65	0.004206	2.19E-125
Q(1,1)	0.015835	0.026962	0.587311
Q(2,2)	0.000659	0.691802	0.000953
Q(3,3)	0.000440	0.757571	0.000581
constant	0.066396	0.004206	15.78748
Final ξ_1	0.111234	0.162426	0.684828
Final ξ_2	-0.029184	0.042777	-0.682235
Final ξ_3	-0.028425	0.052550	-0.540908

sample period (adjusted): 1970 to 1998

$\bar{R}^2 = 1.0000$

s.e. = 7.62E-15

BJ (normality) = 0.018052 [0.991]

LB(1)=0.0496 [0.824]

LB(2)=5.4780 [0.065]

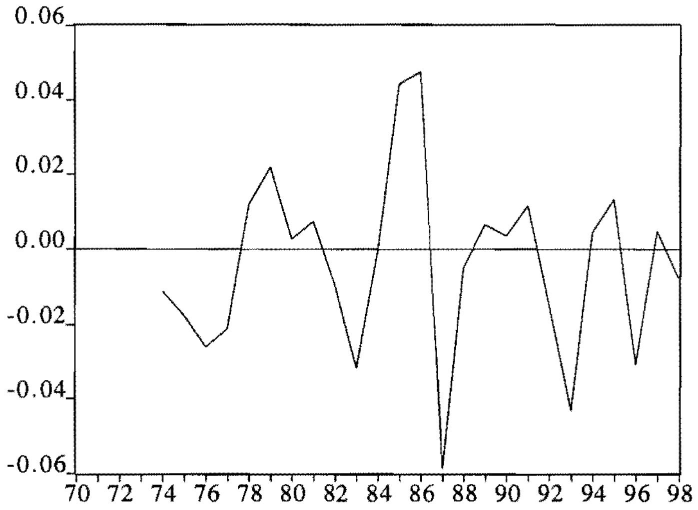
LB(3)=6.1115 [0.106]

LB(4)=6.1125 [0.191]

Note: ξ_1, ξ_2, and ξ_3 are the time-varying coefficients of Δcpi_{t-2}, Δrl_{t-2} and $\Delta exch_{t-2}$, respectively (all variables in natural logarithmic form).

Figure 5.3 is a representation of the one-step-ahead forecast error, that is the residual of the observation equation. Residual diagnostics reported in Table 5.6 indicate adherence to the normality assumption and there is no serial correlation present. The regression statistics (e.g. the R-squared) for time-varying coefficient models are computed by EViews, using the smoothed residuals $y_t - A'x_t - H'\xi_{t|T}$ from the observations equation. The R^2 and adjusted R^2 are equal to unity in this instance.

Figure 5.3 Residual: one-step ahead forecast error graph



Each of the estimated time-varying slope coefficients, assumed to evolve as a random walk with drift, displays a reasonable degree of variation over the period. The intercept coefficient of the expectations rule was not allowed to vary. Figures 5.4 to 5.6 illustrate the evolution of the time-varying coefficients.

The time-varying coefficient of lagged prices clearly displays the largest degree of variation. This coefficient varies roughly between 1.0 and 0.0, with an upward trend between 1974 and 1986, coinciding with a period of rapid increase in the general domestic price level. Double-digit inflation figures were first recorded in 1974 and ascribed mainly to the oil price shocks experienced in 1973. This was repeated in 1979. The high inflation figures during this period reflected the world-wide upward trend in general price levels. Although the main trading partners of South Africa managed to curb their domestic inflation during the 1980s, the upward trend in South African inflation continued. Inflation peaked in 1986 at 18.4 per cent, a period in South African history dominated by international diplomatic and economic isolation. Since 1986 onwards, there has been a general downward tendency in inflation in South Africa, slowing down to a single digit from 1993 onwards. This process was aided by consistently strict monetary policy.

It is interesting to observe that the evolution of the time-varying coefficient of the lagged price level in the price expectations rule to a certain extent mimicked the trend of the actual change in price levels. First, an overall increasing trend emerged from 1974 to 1986 (after an initial downward adjustment of the coefficient), followed by a descent from a high of 0.8 in 1986 to a final value of 0.11 in 1998 when the lowest inflation rate in 28 years was recorded in South Africa, namely 6.8 per cent. What can be gathered from the above may be that consumers are indeed learning from new information as time progresses. In periods of high and rapidly increasing price levels, consumers continuously adjust the parameters of the rule upwards and, as soon as they realise that price levels are declining, they start adjusting their parameters downward, leading to lower expected price levels.

The coefficient of the second variable in the information set of the price expectations rule displays a considerably smaller degree of variation, especially when taking into account that the scale in Figure 5.5 is only 60 per cent of that of Figure 5.4. The coefficient evolves around 0.04 within the band (-0.02, 0.12). The general trend is slightly upward for the period 1974 to 1985, when interest rates peaked at 18.62 per cent, followed by a movement sideways in line with interest rate movements.

The variation in the coefficient of the lagged spot nominal rand/US dollar exchange rate is also significantly less than that of lagged prices. The coefficient varies around –0.016, between -0.089 and 0.017. The coefficient evolves rather smoothly, with only one significant jump in 1985, a year marked by the largest depreciation in the rand/US dollar exchange rate in history, namely a depreciation of 50 per cent from R1,48/US dollar to R2,23/US dollar. This pronounced depreciation may clearly be attributed to political and economic isolation, the debt standstill and disruptive balance of payments adjustments.

Figure 5.4 Coefficient on lagged consumer price inflation

Figure 5.5 Coefficient on lagged long-term interest rate

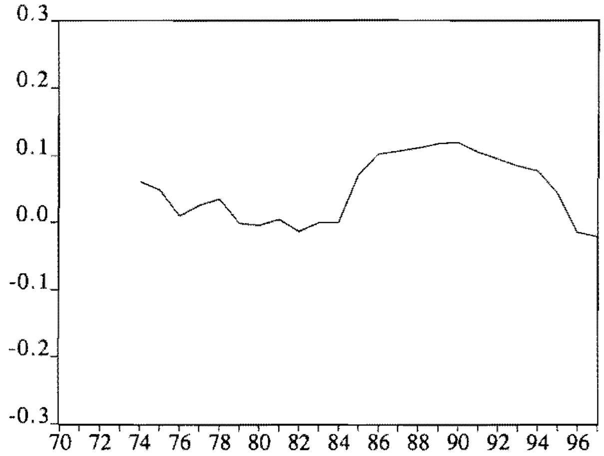
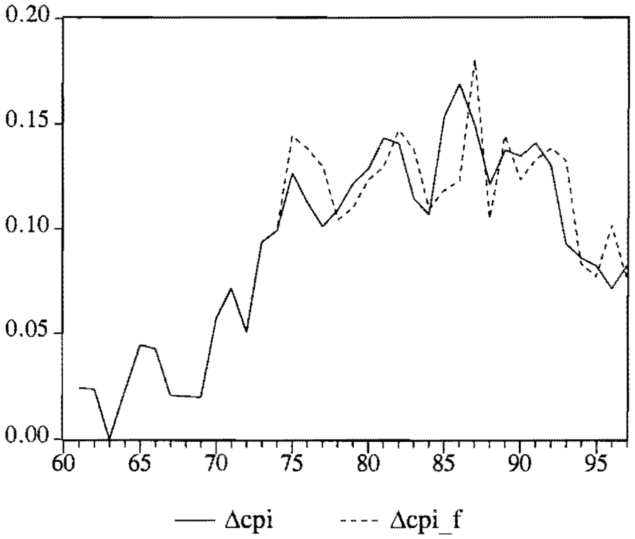


Figure 5.6 Coefficient on lagged rand/US dollar exchange rate

It is obvious from the above that the most important variable in the information set of the price expectations rule is the lagged price level. The other two variables display a lesser degree of variation and are also statistically less significant in explaining price expectations. Furthermore, the larger variance of the one-period-ahead forecast of the first element of the state vector, namely that of the lagged price level, is also indicative of a faster rate of learning with respect to observed changes in the actual price level than that of the other two independent variables.

The actual price level and its Kalman filter prediction for the specification $\Delta cpi_t = f(\Delta cpi_{t-2}, \Delta rl_{t-2}, \Delta exch_{t-2})$ are shown in Figure 5.7.

Figure 5.7 Kalman filter estimation of expected price level: Δcpi and Δcpi_f



In order to implement the price expectations variable in the consumption functions, the above was repeated for the specification

$$\Delta cpi_{t+1} = f(\Delta cpi_{t-1}, \Delta rl_{t-1}, \Delta exch_{t-1}) \tag{5.10}$$

or put differently,

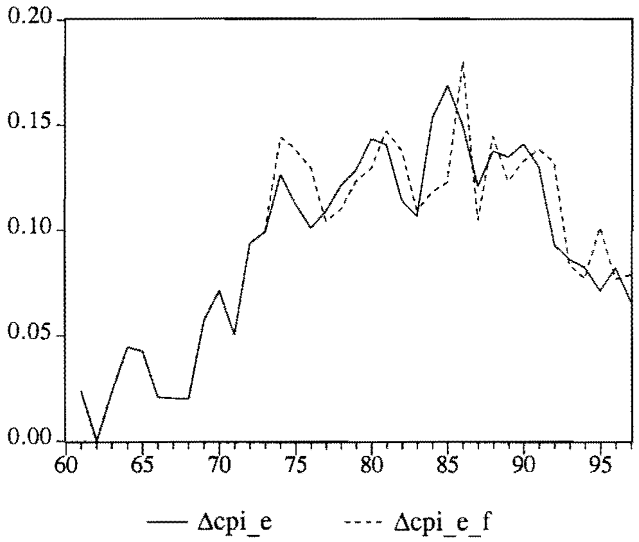
$$\Delta cpi\_e_t = f(\Delta cpi\_e_{t-2}, \Delta rl_{t-1}, \Delta exch_{t-1}) \tag{5.11}$$

with $\Delta cpi\_e_t = \Delta cpi_{t+1}$.

The reason for implementing the price expectations rule in the form of (5.11), is that the software package used to dynamically solve the consumption expenditure model is unable to determine a solution for the dependent variable with Δcpi_{t+1} included in the information set.

The estimation result of model (5.11) corresponds exactly to that reported in Table 5.6, with the exception that the adjusted sample period is from 1969 to 1997 instead of from 1970 to 1998. The graphical representation of the actual and fitted values for Δcpi_e_t in Figure 5.8 confirms this fact. The fitted value, $\Delta cpi\_e\_f_t$, is therefore the variable to be implemented in the consumption functions as the variable reflecting the price expectations of the consumer. Citing Cuthbertson (1988:226) in this regard: "It is worth noting at the outset that an expectations series generated using the Kalman filter with time-varying parameters may be directly used in behavioural equations where the structural parameters are assumed constant."

Figure 5.8 Kalman filter estimation of expected price level: Δcpi_e and Δcpi_e_f



5.5.2 Forward-looking consumption functions

Finally, the behavioural equations can be estimated, containing the one-period-ahead forecast of the expectations variable as part of the short-run dynamics of the system; that is the actual result obtained through the Kalman filter estimation.

The long-run relationship between private consumption expenditure, wealth and personal disposable income, in the case of total private consumption expenditure and durable consumption expenditure, was estimated in the form c-yd = f(w-yd), thus constraining the sum of wealth and income elasticities to unity (all variables in natural logarithm form), for the specification

$$c_t = Ayd_t^{\alpha} w_t^{\beta} e^{\varepsilon_t} \qquad \text{with } 0<\beta<1;\ 0<\alpha<1;\ \alpha+\beta=1. \qquad (5.12)$$

In the case of non-durable consumption, only current disposable consumption is included in the long-run equilibrium equation, with an expected income elasticity close to unity. Interest rates are believed to play a significant role in the explanation of the durable consumption expenditure category and, since durable consumption (including durables and non-durables) constitutes 27 per cent of total consumption, possibly also in the total private consumption expenditure function. Interest rates are, however, not included in the long-run equilibrium equation, but rather considered as contributing towards the short-run dynamic structure of the system.

Since expenditure on durables accounts only for 11 per cent and semi-durables for 16 per cent of total private consumption expenditure, the durable consumption function was estimated on the aggregate of the two components.

In all instances, a vector autoregressive (VAR) model with a lag length of 1 was used to test for the number of cointegrating relationships. Although the Akaike information criterion (AIC) and the Schwarz Bayesian criterion (SBC) suggested that a model with lag length 2 should be selected, the values of the information criteria in all cases were very close for the two models. Granger (1991:561) also notes in this regard that the AIC criterion is known to 'overfit' in the sense that if the true value of p for VAR(p) is p_0 (finite), then the criterion is inclined to choose a value for p that is somewhat greater than p_0. The validity of the Johansen estimation technique depends on white noise error terms – the minimum lag length that renders residuals of the ECM white noise would thus be acceptable. In all instances, the error correction models passed all the standard tests of serial correlation, normality, etc. where long-run coefficients were obtained with a VAR of order 1. The limited sample of the data from 1970 to 1997 (constrained by the unavailability of the financial wealth variable) and the fact that the Johansen cointegration test is rather data-hungry, contributed towards the decision to use a VAR with lag length 1 throughout. The resulting long-run coefficients, in most instances, turned out to be very similar for a VAR(1) and a VAR(2). For all behavioural equations, values for the AIC and SBC as well as the long-run coefficients obtained with a specified lag length of 1 and 2, respectively, are reported.

The results of individual stochastic functions are subsequently reported in sections 5.5.2.1 to 5.5.2.3. Three possible error correction specifications for durable consumption expenditure are presented. The selection of the best model, based on model selection criteria and economic theory, is motivated.

5.5.2.1 Total private consumption expenditure

The long-run relationship between total private consumption expenditure, wealth and personal disposable income was estimated in the form c-yd = f(w-yd), thus constraining the sum of wealth and income elasticities to unity (all variables in natural logarithmic form). Both wealth variables were tested, namely the stock of wealth, w, and the return on wealth, rw. In this instance, total private consumption expenditure, the return on wealth and personal disposable income constituted a cointegrating relationship.

(i) The cointegration equation

In order to test for cointegration between (c-yd) and (rw-yd), the Johansen test was employed. The correct lag length for the VAR must be selected to ensure that the error correction model has Gaussian errors. Both AIC and SBC suggest that a VAR model with a lag length of 2 should be used. The result of these measures is reported in Table 5.7.

Table 5.7 Selection of the order of the VAR: Total private consumption expenditure

Order of VAR	AIC	SBC
1	57.2617	55.3180
2	53.7687	50.6234

The values of the test statistics proved to be close for the two models. Since the sample contains only 27 observations and seeing that the Johansen procedure is rather data-hungry, the cointegration test was conducted for a VAR of both length 1 and 2.

In the case of a VAR with lag length 2, cointegration could only be established at a 10 per cent significance level. For a VAR with lag length equal to 1, the Johansen likelihood ratio test for the number of cointegrating relationships (denoted by r), based on the maximum eigenvalue and the trace of the stochastic matrix, suggests one cointegrating relationship at a 5 per cent significance level between the variables in the long-run relationship (the cointegration equation). This result is presented in Table 5.8.

Table 5.8 Johansen test for the number of cointegrating relationships: total private consumption expenditure

Cointegration LR test based on maximal eigenvalue of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	r=1	16.8664	15.8700	13.8100
$r \leq 1$	r=2	7.0921	9.1600	7.5300
Cointegration LR test based on trace of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	$r \geq 1$	23.9585	20.1800	17.8800
$r \leq 1$	r=2	7.0921	9.1600	7.5300

Note: VAR order = 1

The long-run relationship was estimated with an unrestricted constant and the long-run cointegration relationship, for rank equal to 1 (r=1), is reported in Table 5.9 (figures in brackets denote coefficients normalised on the dependent variable).

Table 5.9 Estimate of cointegration equation

Dependent variable: $(ctot - yd)_t$

Variables	Cointegrating vector
$(ctot-yd)_t$	4.4839 (-1.0)
$(rw-yd)_t$	-0.88128 (0.19655)
constant	-2.5077 (0.55926)
Sample Period: 1971-1997 Order of VAR = 1; r=1	

The above result suggests the presence of a long-run relationship of the form

$$ctot_t = (1\text{-}0.19655)yd_t + (0.19655)rw_t + 0.55926. \qquad (5.13)$$

The income elasticity therefore amounts to 0.80345 and the wealth elasticity to 0.19655 for the period under consideration. (For a VAR(2), the wealth elasticity amounts to 0.17302 and the intercept coefficient equals 0.48837.) Coefficients are of the correct sign and the magnitudes correspond with *a priori* expectations expressed in the hypothesised long-run relationship.

The residual derived from the above, allows for the specification of the error correction model, representing the short-run dynamic adjustment process. This result is reported in Table 5.10.

(ii) The error correction model

Both measures of non-human wealth were introduced in the specification of total private consumption expenditure. While the return to wealth is utilised in the long-run cointegration relationship, the actual real stock of wealth seems to contribute to the short-run dynamics of the system, so do lagged values of wealth stock and current real disposable income, lagged total consumption expenditure and the future expected price level. Lagged values of the dependent variable, total private consumption expenditure, would account for habit persistence or historic expenditure patterns, tastes of the consumer, etc. The expected price level, that is the one-period-ahead Kalman filter estimation, is introduced to test the hypothesis that consumers are forward-looking with respect to prices. Al variables are used in natural logarithmic form.

Diagnostic statistics suggest that the equation is statistically well-specified, with no violations of the Gaussian assumption. The use of a VAR of order 1 for the derivation of the long-run relationship, is therefore justified. Figures in square brackets denote probability values (the probability of falsely rejecting the null hypothesis of a zero restriction on the coefficient or diagnostic). BJ denotes the Bera-Jarque test for the normality assumption; LB and LM denote tests for serial correlation; Ramsey's RESET is a test for misspecification and ARCH and White denote tests for the homoskedasticity assumption. All test statistics are distributed χ^2, with figures in round brackets denoting degrees of freedom. Recursive estimates also confirmed stability in the parameters.

The result of *ex post* simulation of the private consumption function is presented graphically in Figure 5.9. The fitted values also constitute the base-line forecast against which the response characteristics of the system to exogenous income and wealth shocks are analysed.

Table 5.10 An error correction model for total private consumption expenditure

Dependent variable: $\Delta ctot_t$

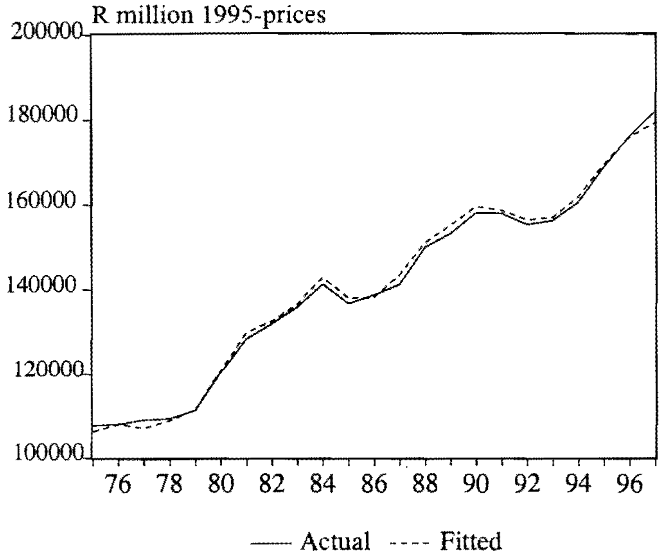
Variable	Coefficient	Std. Error	t-Statistic
constant	0.040825	0.011592	3.521846
$resid_{t-1}$	-0.205334	0.070992	-2.892357
$\Delta ctot_{t-1}$	-0.235168	0.105383	-2.231562
Δyd_t	0.670364	0.067966	9.863180
Δw_t	-0.316304	0.040438	-7.821941
Δw_{t-1}	0.120829	0.032498	3.717989
Δcpi^*_{t+1}	-0.216683	0.088190	-2.456994

sample period: 1975 to 1997
R^2 = 0.909710
$\bar{R}^2$ = 0.875851
s.e. = 0.009458

Normality:	BJ(2)=0.7026	[0.7026]
Serial correlation:	LB(8)=3.4096	[0.9096]
	LM(2)=2.9052	[0.2340]
Heteroscedasticity:	ARCH(1)=0.0415	[0.8385]
	White(1)=6.6778	[0.8781]
Stability:	RESET(2)=0.7714	[0.6799]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter prediction of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

Figure 5.9 The overall dynamic fit of the total private consumption expenditure function



5.5.2.2 Durable private consumption expenditure

The long-run relationship between durable private consumption expenditure, wealth and personal disposable income was estimated in the form cdur-yd = f(w-yd), thus constraining the sum of wealth and income elasticities to unity (all variables in natural logarithmic form). Both wealth variables were tested, namely the stock of wealth, w, and the return to wealth variable, rw. In the case of durable private consumption expenditure, durable consumption, the stock of financial wealth and personal disposable income constituted a cointegrating relationship.

(i) The cointegration equation

In order to test for cointegration between (cdur-yd) and (w-yd), the Johansen test was employed. As in the case of total private consumption expenditure, both AIC and SBC suggest that a model with a lag length of 2 should be used. The information criteria are reported in Table 5.11.

Table 5.11 Selection of the order of the VAR: Durable private consumption expenditure

Order of VAR	AIC	SBC
1	45.2640	43.3203
2	42.4419	39.2966

The values of the information criteria prove to be very close for the two models. Since the sample contains only 27 observations, the cointegration test was conducted for both a VAR of length 1 and 2.

As with previous results, in the case of a lag length of 2, cointegration could not be established at either a 5 or a 10 per cent significance level. For a VAR with lag length equal to 1, the Johansen likelihood ratio test for the number of cointegrating relationships, based on the maximum eigenvalue and the trace of the stochastic matrix, suggests the presence of one cointegrating relationship at a 5 per cent significance level. These results are presented in Table 5.12.

Table 5.12 Johansen test for the number of cointegrating relationships: durable private consumption expenditure

Cointegration LR test based on maximal eigenvalue of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	r=1	24.2811	15.8700	13.8100
r ≤ 1	r=2	2.1750	9.1600	7.5300
Cointegration LR test based on trace of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	r ≥ 1	26.4561	20.1800	17.8800
r ≤ 1	r=2	2.1750	9.1600	7.5300

Note: VAR order = 1

The long-run cointegration relationship is presented in Table 5.13. With respect to the inclusion of deterministic variables in the long-run equation, including a constant, but no trend, yielded economically viable results. Figures in brackets denote coefficients normalised on the dependent variable.

Table 5.13 Estimate of cointegration equation

Dependent variable: $(cdur - yd)_t$

Variables	Cointegrating vector
(cdur-yd)t	-3.0301 (-1.0)
$(w\text{-}yd)_t$	0.72817 (0.24031)
constant	-3.2266 (-1.0648)
Sample Period: 1971-1997 Order of VAR = 1; r=1	

The above result suggests the presence of a long-run relationship of the form

$$cdur_t = (1\text{-}0.24031)yd_t + (0.24031)w_t - 1.0648. \qquad (5.14)$$

The income elasticity therefore amounts to 0.75969 and the wealth elasticity to 0.24031 for the period under consideration. The signs and magnitudes of coefficients correspond to *a priori* expectations. (Although cointegration could not be proved for a VAR model with lag length 2, the slope coefficient of the relationship for r=1 in this instance turned out to be equal to 0.25208 and the intercept coefficient to be equal to -1.0484).

The residual derived from the above, allows for the specification of the error correction model, representing the short-run adjustment to equilibrium reported in Table 5.14. Two alternatives are also presented, reported in Table 5.15 and Table 5.16.

(ii) The error correction model

Only the stock of wealth variable as a measure of non-human wealth was introduced in the specification of durable private consumption expenditure. Variables contributing to the short-run dynamics of the system include current real disposable income, real financial wealth stock as well as lagged values of this variable, a representative long-term interest rate lagged by one period, relative prices (that is the ratio between prices of durables relative to non-durables) and the future expected price level. The expected price level, that is the one-period-ahead Kalman filter prediction, is introduced to test the hypothesis that consumers are forward-looking with respect to prices when making durable consumption expenditure decisions. Al variables are used in natural logarithmic form. Since the intercept coefficient had a low t-statistic (t=1.33) in the original estimation, signalling statistical insignificance, it was omitted from the equation, with the following result.

Diagnostic statistics suggest that the equation is statistically well-specified, and that the use of a VAR with lower length than was selected by the information criteria, was justified in this case, since the errors of the ECM adhere to the Gaussian assumption. Figures in square brackets denote probability values (the probability of falsely rejecting the null hypothesis of a zero restriction on the coefficient or diagnostic). BJ denotes the Bera-Jarque test for the normality assumption; LB and LM denote tests for serial correlation; Ramsey's RESET is a test for misspecification and ARCH and White denote tests for the homoskedasticity assumption. All test statistics are distributed χ^2, with figures in round brackets denoting degrees of freedom. Recursive estimates also confirmed stability in the parameters.

The result of *ex post* simulation of the durable private consumption function is presented graphically in Figure 5.10.

Table 5.14 An error correction model for durable private consumption expenditure

Dependent variable: $\Delta cdur_t$

Variable	Coefficient	Std. Error	t-Statistic
$resid_{t-1}$	-0.339951	0.089959	-3.778934
Δyd_{t-1}	1.206395	0.111066	10.86197
Δw_t	-0.547395	0.057194	-9.570830
Δw_{t-1}	0.252309	0.070234	3.592415
Δrl_{t-1}	-0.096311	0.031806	-3.028070
$\Delta relp_{t-1}$	-0.458177	0.126838	-3.612293
Δcpi^*_{t+1}	-0.082389	0.039528	-2.084328

sample period (adjusted): 1975 to 1997

$R^2 = 0.938790$

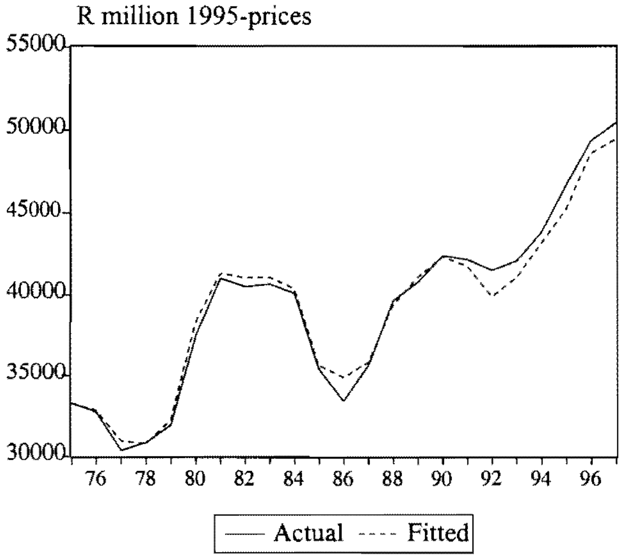
$\bar{R}^2 = 0.915837$

s.e. = 0.017725

Normality:	BJ(2)=0.7912	[0.6733]
Serial correlation:	LB(8)=6.6335	[0.5570]
	LM(2)=2.4212	[0.2980]
Heteroscedasticity:	ARCH(1)=0.0182	[0.8927]
	White(1)=19.1092	[0.1608]
Stability:	RESET(2)=2.3362	[0.3110]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter estimation of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

Figure 5.10 The overall dynamic fit of the durable private consumption expenditure function



The intercept term was suppressed in the above case. Although it is generally preferable to include unrestricted constants in the ECM, even if not statistically significant, it is also considered acceptable to restrict the constant in the ECM in order to obtain a more reasonable model. Davidson, Hendry, Srba and Yeo (1978:687) did this in their 1978 paper on consumption in order for the error correction model to work (see equation (2.16)). However, the dynamic simulation of the equation seems to perform poorly over the last number of observations, constantly underestimating the level of durable consumption expenditure.

It must be pointed out that even if the unrestricted constant is retained, the results are acceptable. This result is reported in Table 5.15, with the overall fit of this specification represented in Figure 5.11. The *ex post* simulation, in this instance, yields a satisfactory in-sample fit.

One may argue that the p-value of 0.2 associated with the t-value of 1.339 of the constant in Table 5.15, although not significant, can still be in an ambiguous range and that bias may be introduced by restricting the constant to zero.

Table 5.15 An error correction model for durable private consumption expenditure, containing an unrestricted constant

Dependent variable: $\Delta cdur_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	0.029138	0.021748	1.339805
$resid_{t-1}$	-0.280156	0.092791	-4.096925
Δyd_{t-1}	1.179692	0.110222	10.70286
Δw_t	-0.559272	0.056523	-9.89457
Δw_{t-1}	0.232467	0.070133	3.314681
Δrl_{t-1}	-0.080064	0.033328	-2.402260
$\Delta relp_{t-1}$	-0.44224	0.126367	-3.357082
Δcpi^*_{t+1}	-0.308901	0.173410	-1.781335

sample period (adjusted): 1975 to 1997

R^2 = 0.945332

$\bar{R}^2$ = 0.919821

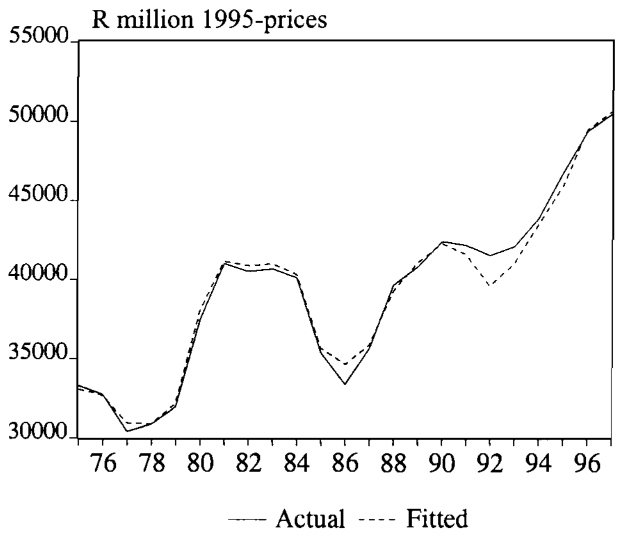
s.e. = 0.017300

Normality:	BJ(2)=1.7408	[0.4188]
Serial correlation:	LB(8)=5.9891	[0.6480]
	LM(2)=2.5668	[0.2771]
Heteroscedasticity:	ARCH(1)=0.0982	[0.7540]
	White(1)=15.3248	[0.3563]
Stability:	RESET(2)=0.9783	[0.6131]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter estimation of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

The motivation for restricting the constant of the ECM in the first place, was because a t-value of the price expectations variable of 1.78 in absolute terms (with an associated p-value of 0.0951), may not be significant enough to make a strong case in favour of the effect of price expectations on durable consumption expenditure decisions. However, a p-value of less than 0.10 may often be considered significant 'enough'. When performing a two-tailed test on the coefficient of Δcpi^*_{t+1}, the cut-off value of 10 per cent significance with 15 degrees of freedom would be -1.753. This perhaps provides more evidence on the significance of the relationship between price expectations and durable consumption. Since the signs (and magnitudes) of the coefficients in the ECM are generally not interpreted, a one-tailed test (that yields a critical value of 1.753 on a 5 per cent significance) would not be appropriate in this instance.

Figure 5.11 The overall dynamic fit of the durable private consumption expenditure function, containing an unrestricted constant



As a last option, if one is prepared to accept that the South African consumer is not interest rate-sensitive in making durable consumption expenditure decisions, the ECM reported in Table 5.16, may also be regarded as an acceptable structural explanation of durable consumption expenditure behaviour in South Africa.

Table 5.16 An error correction model for durable private consumption expenditure, excluding the interest rate variable from the information set

Dependent variable: $\Delta cdur_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	0.048148	0.023081	2.086034
$resid_{t-1}$	-0.482087	0.094022	-5.127377
Δyd_{t-1}	1.176291	0.125574	9.367303
Δw_t	-0.523292	0.062099	-8.426698
Δw_{t-1}	0.168625	0.070948	2.280311
$\Delta relp_{t-1}$	-0.387292	0.142910	-2.710045
Δcpi^*_{t+1}	-0.470578	0.182092	-2.584289

sample period (adjusted): 1975 to 1997

$R^2 = 0.924301$

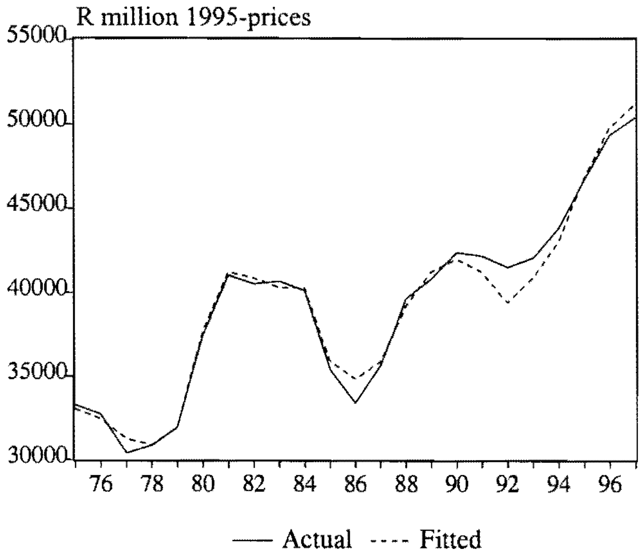
$\bar{R}^2 = 0.895913$

s.e. = 0.019711

Normality:	BJ(2)=1.0547	[0.9730]
Serial correlation:	LB(8)=6.4233	[0.6000]
	LM(2)=0.0349	[0.9828]
Heteroscedasticity:	ARCH(1)=0.0524	[0.8189]
	White(1)=14.895	[0.2474]
Stability:	RESET(2)=0.4219	[0.8097]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter estimation of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

Figure 5.12 The overall dynamic fit of the durable private consumption expenditure function, excluding the interest rate variable from the specification



Based on diagnostic statistics, all three models appear to be statistically well specified. The dynamic fit of the first option, where the intercept coefficient was restricted to zero, was less than satisfactory. The third option, suggesting interest rate insensitivity on the part of the consumer, does not correspond with economic theory which states that, within the intertemporal optimisation framework, interest rates constitute a trade-off between current and future consumption. On this basis, interest rates certainly should play a role, especially in explaining durable consumption. On these grounds, the second option may be considered optimal. Model selection criteria, namely the adjusted R-squared statistic and the Akaike information criterion support this decision while the Schwarz Bayesian criterion favours the model with restricted intercept. This result is reported in Table 5.17.

The model of which the overall dynamic fit is depicted in Figure 5.11 will therefore be regarded as the baseline forecast for durable consumption against which the response characteristics of the system to exogenous shocks will be analysed. This result is reported in section 5.8.

Table 5.17 Model selection criteria for the durable consumption expenditure function

Model	Adjusted R^2	AIC	SBC
Option 1: Restricted intercept	0.915837	-4.981926	-4.636341
Option 2: Unrestricted intercept	0.919821	-5.008005	-4.613051
Option 3: Exclusion of interest rates	0.895913	-4.769461	-4.423876

5.5.2.3 Non-durable private consumption expenditure

A long-run relationship was established between non-durable consumption expenditure and personal disposable income in the form c = f(yd), with variables in natural logarithmic form. No restrictions were placed on the income elasticity, but it is expected to be close to unity. The series for non-durable consumption expenditure and personal disposable income also constituted a cointegrating relationship.

(i) The cointegration equation

In order to test for cointegration between cndur and yd, the Johansen cointegration test was employed. First, the AIC and SBC were used to establish the appropriate lag length of the VAR model, i.e. the minimum lag length, which would ensure Gaussian errors for the error correction model. Both AIC and SBC suggest that a model with a lag length of 2 should be used. The result of these measures is reported in Table 5.18.

The values of the test statistics prove to be close for the two models. For reasons argued before, the cointegration test was conducted for both a VAR(1) and a VAR(2) model.

Table 5.18 Selection of the order of the VAR: Non-durable private consumption expenditure.

Order of VAR	AIC	SBC
1	67.3497	66.0175
2	67.0948	64.4304

For a VAR with lag length equal to 1, the Johansen likelihood ratio test for the number of cointegrating relationships, based on the maximum eigenvalue and the trace of the stochastic matrix, suggests one cointegrating relationship at a 5 per cent significance level between the variables in the long-run relationship (the cointegration equation). This result is presented in Table 5.19 and the long-run cointegration relationship in Table 5.20. The specification, with constant and trend restricted to zero (no deterministic variables included in long-run relationship) yielded results in line with those hypothesised.

Table 5.19 Johansen test for the number of cointegrating relationships: non-durable private consumption expenditure

Cointegration LR test based on maximal eigenvalue of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	r=1	32.1893	11.0300	9.2800
r ≤ 1	r=2	3.5239	4.1600	3.0400
Cointegration LR test based on trace of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	r ≥ 1	35.7132	12.3600	10.2500
r ≤ 1	r=2	3.5239	4.1600	3.0400

Note: VAR order = 1

Table 5.20 Estimate of cointegration equation

Dependent variable: $cndur_t$

Variables	Cointegrating vector
$cndur_t$	-1.3686 (-1.0)
yd_t	1.2861 (0.93969)
Sample Period: 1970-1997 Order of VAR = 1; r=1	

The above result suggests the presence of a long-run relationship of the form

$$cndur_t = 0.93969yd_t . \qquad (5.15)$$

The income elasticity therefore amounts to 0.93969, which is relatively close to unity, for the period under consideration. (For a VAR model with lag length of 2, a cointegrating relationship with income elasticity equal to 0.93969 was established, a result almost identical to the reported result of a VAR with a lag length of 1.)

The residual derived from the above, allows for the specification of the error correction model, reported in Table 5.21.

(ii) The error correction model

Variables contributing to the short-run dynamics of the system include current real disposable income, real financial wealth stock as well as relative prices (that is the ratio between prices of durables and non-durables) and a variable reflecting labour market conditions, namely the employment rate in the non-agricultural sector. The future expected price level was not significant in the explanation on non-durable consumption expenditure, nor did interest rates prove to be statistically significant. All variables were used in natural logarithmic form. The constant had a low t-score (t=-0.81), signalling statistical insignificance, but could not be dropped form the equation for the following reason. The specification should always include a constant in one form or another; just for scaling of the data, a constant should be included for the cointegrating vector. If the data has non-zero growth, this may be coming from an exogenous variable or from a drift term (constant) in an unrestricted ECM equation. The constant was therefore retained in this instance.

Diagnostic statistics suggest that the equation is statistically well-specified, and that a VAR with lag length of 1 was sufficient to render a white noise error term for the error correction model. Figures in square brackets denote probability values (the probability of falsely rejecting the null hypothesis of a zero restriction on the coefficient or diagnostic). BJ denotes the Bera-Jarque test for the normality assumption; LB and LM denote tests for serial correlation; Ramsey's RESET is a test for misspecification and ARCH and White denote tests for the homoskedasticity assumption. All test statistics are distributed χ^2, with figures in round brackets denoting degrees of freedom. Recursive estimates confirmed stability in the parameters.

The result of *ex post* simulation of the private non-durable consumption function is presented graphically in Figure 5.13. The fitted values once again constitute the base-line forecast against which the response characteristics of the system to exogenous income and wealth shocks will be analysed.

Table 5.21 An error correction model for non-durable private consumption expenditure

Dependent variable: $\Delta cndur_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	-0.006294	0.007789	-0.808036
$resid_{t-1}$	-0.223873	0.059900	-3.737458
Δyd_t	0.272376	0.086796	3.138119
Δw_t	-0.102052	0.033484	-3.047824
$\Delta relp_t$	0.260865	0.085547	3.049375
$\Delta emplna_t$	0.373183	0.136430	2.735335

sample period (adjusted): 1971 to 1997

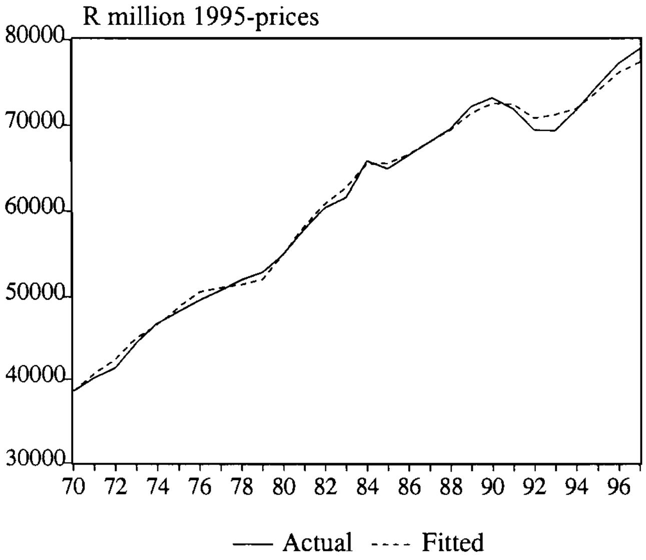
R^2 = 0.753823

$\bar{R}^2$ = 0.695209

s.e. = 0.013030

Normality:	BJ(2)=2.2545	[0.3239]
Serial correlation:	LB(8)=5.0631	[0.7510]
	LM(2)=1.3352	[0.5129]
Heteroscedasticity:	ARCH(1)=0.6945	[0.4046]
	White(1)=9.8576	[0.4531]
Stability:	RESET(2)=6.9559	[0.0309]

Figure 5.13 The overall dynamic fit of the non-durable private consumption expenditure function



5.6 LONG-RUN RELATIONSHIPS: COMPARISON OF RESULTS OBTAINED FROM JOHANSEN AND ENGLE-GRANGER PROCEDURES

In order to prevent cluttering of the reported results obtained for long-run relationships between consumption, income and wealth variables, results obtained with the Engle-Granger estimation technique are compared separately in this section with results obtained with the Johansen approach. These results are documented in Table 5.22. The Engle-Granger equilibrium relationships were not incorporated into error correction models, thus the third-step correction as suggested by Engle and Yoo (1989) was not conducted. EG-cointegration denotes the ADF unit root test statistic, which should be evaluated against the MacKinnon (1991:275) response surface critical values.

The results contained in Table 5.22 confirm the discussion in section 4.4.3, stating that even if a single cointegrating vector exist between variables in the long-run relationship, the results obtained with a single equation estimation technique would differ from the result obtained with the Johansen technique. The reason for this is, as stated before, that the Engle-Granger approach effectively ignores the short-run dynamics when estimating the long-run equilibrium relationship. Another reason for the difference in outcome pertains to the endo-exogenous division of variables. Any simultaneity between variables is disregarded in a single equation estimation approach. Charemza and Deadman (1997:150) noted in this regard that a rise in income (or wealth) would lead to a rise in consumption. Due to the income identity, however, it would be impossible to change the value of consumption without influencing income (or wealth). All three variables would thus be regarded as endogenous variables and be described as jointly dependent variables.

Table 5.22 Comparison of long-run relationships between the Johansen approach and the Engle-Granger procedure

Categories of consumption	**Johansen**	**Engle and Granger**
Total	$ctot_t = (1\text{-}0.19655)yd_t$ $+(0.19655)rw_t$ $+0.55926$	$ctot_t = (1\text{-}0.06699)yd_t$ $+(0.06699)rw_t$ $+0.15083$ EG-cointegration: -3.2710
Durable	$cdur_t = (1\text{-}0.24031)yd_t$ $+(0.24031)w_t$ -1.06480	$cdur_t = (1\text{-}0.47729)yd_t$ $+(0.47729)w_t$ -1.71432 EG-cointegration: -3.2013
Non-durable	$cndur_t = 0.93969yd_t$	$cndur_t = 0.928874yd_t$ EG-cointegration: -2.0592 *or* $cndur_t = 1.03218yd_t$ -1.22256 EG-cointegration: -2.4713

5.7 THE MODEL SOLUTION

Since stochastic estimation of total consumption is believed to be more reliable than that of consumption expenditure on services, expenditure on services was deterministically determined as the residual of the total and the other two categories. The quality of the in-sample fit is presented below as measured by the root-mean-squared-percentage-error (Klein *et al.* 1983:76):

$$RMSPE = \sqrt{\sum_{t=1}^{T}[(y_t - \hat{y}_t)/y_t]^2 / T} \times 100 . \qquad (5.16)$$

As expected, the root-mean-squared-error value for services is higher than for the categories stochastically estimated, but the measure is still within the 5 per cent acceptable range (*op. cit.*:76).

Table 5.23 RMSPE for behavioural equations

Private consumption expenditure categories			
Total	**Durable**	**Non-durable**	**Services**
0.81%	1.80%	1.15%	4.24%

5.8 RESPONSE CHARACTERISTICS OF ERROR CORRECTION MODELS

Deterministic analysis of the response characteristics of the model was conducted to test whether the short and long-run response characteristics correspond to theoretical priors and long-run equilibrium properties of the data. The process consists of conducting a dynamic baseline forecast for each behavioural equation. An exogenous shock is then applied to the income variable and subsequently to the wealth variable(s), and the adjustment path towards a new equilibrium outcome is determined.

A shock of 10 per cent was considered to be significant and first, personal disposable income was allowed to rise 10 per cent above its actual level from 1977 onwards. This date was considered appropriate to commence with the exogenous shock, since the adjusted sample period over which the estimation and baseline forecast were conducted was from 1975 to 1997. The same procedure was repeated, applying an exogenous shock of 10 per cent to the financial wealth variable, with the associated effect on return to wealth, from 1977 onwards.

The expected response in total private consumption expenditure as a result of a sustained 10 per cent exogenous shock to personal disposable income would, for example, ultimately mean an adjustment towards a new equilibrium level, 8.03 per cent higher than would be expected without the exogenous shock. A 10 per cent increase in the wealth variable, on the other hand, would only result in a 1.97 per cent increase in total private consumption.

The expected multiplier effects as indicated by the respective elasticity values in the long-run cointegration equations and the actual outcome over the sample period are reported in Table 5.24 and Table 5.25.

Table 5.24 Expected and actual multiplier effect of a 10 per cent exogenous shock to personable disposable income from 1977 onwards

Categories of consumption	Income elasticity	Expected result (% change in consumption)	Final value (% change in consumption)
Total	0.80345	8.0345%	7.8748%
Durable	0.75969	7.5969%	7.5092%
Non-durable	0.93935	9.3935%	9.3223%

Table 5.25 Expected and actual multiplier effect of a 10 per cent exogenous shock to financial wealth from 1977 onwards

Categories of consumption	Wealth elasticity	Expected result (% change in consumption)	Final value (% change in consumption)
Total	0.19655	1.9655%	1.79748%
Durable	0.24031	2.4031%	2.3168%
Non-durable	–	0%	-0.0061%

Both durable and non-durable consumption expenditure categories reach their new equilibrium values (in accordance with the respective long-run coefficients of the cointegration equations) within the sample period. After an initial overshooting, adjustment proceeds along smooth paths towards the new equilibrium levels of expenditure, as illustrated by Figures 5.15 and 5.16. For total consumption, the new expected equilibrium value is not quite reached, but appears to be well within reach. The graphical representation of the adjustment path in Figure 5.14 indicates that the adjustment path is progressing well towards this value. The speed of adjustment is generally determined by the coefficient of the lagged residual of the long-run cointegration equation in the error correction model.

Vertical axes measures the difference in outcome of the baseline simulation and the simulation subjected to the exogenous shock, as percentage of the level of the dependent variable in the baseline outcome.

Figure 5.14 Response characteristics of total private consumption expenditure

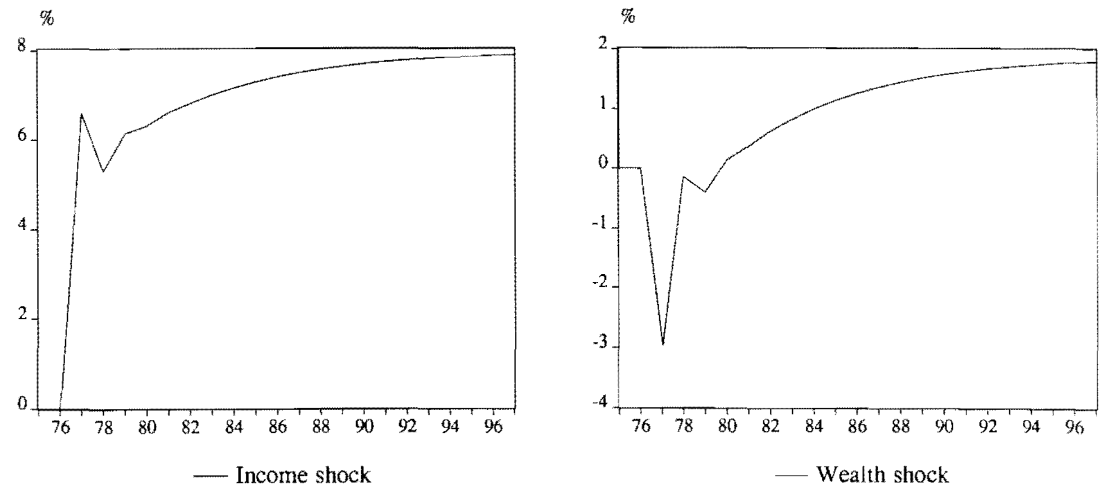


Figure 5.15 Response characteristics of durable private consumption expenditure

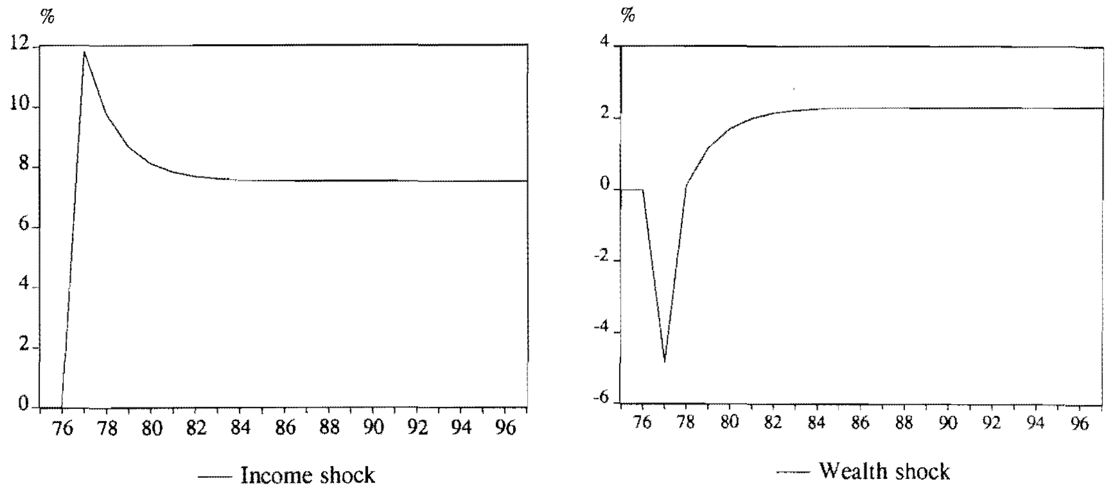
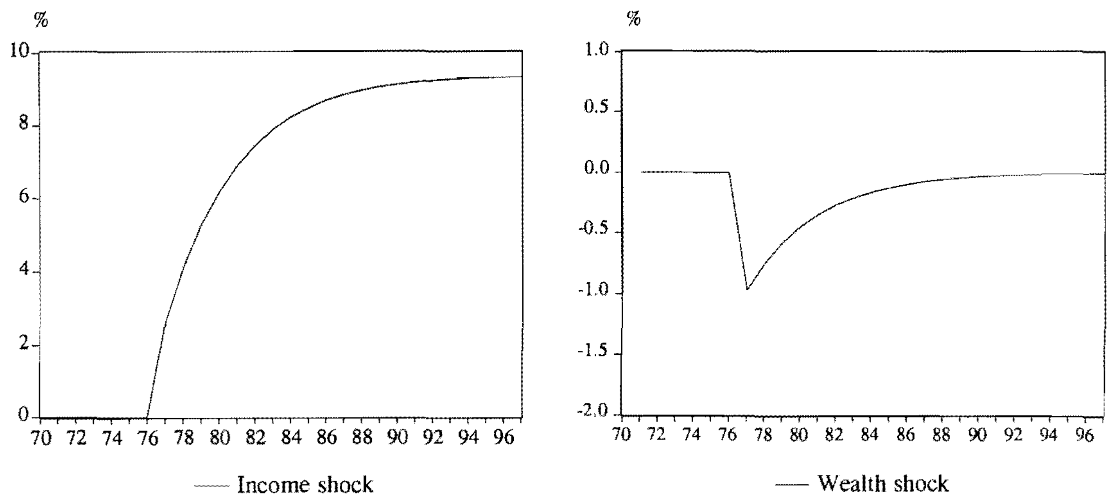


Figure 5.16 Response characteristics of non-durable private consumption expenditure



5.9 CONCLUSION

This chapter tested the hypothesis that South African consumers are forward-looking with respect to prices when making consumption expenditure decisions. Furthermore, it was also tested whether the assumption that consumers learn through a boundedly rational learning process from expectations errors made in the past and that they find a way of increasing their knowledge about the true values of the parameters in the expectations rule, is correct.

The expectations rule was formulated in an attempt to capture the psychological learning process of intelligent economic agents. Variables influencing price expectations were taken to be the lagged observed price level, lagged interest rates and the lagged exchange rate. The most important variable in the information set proved to be the lagged price level. The time-varying coefficient of this variable also displayed the largest variation with an upward tendency in periods with high and increasing price levels and *visa versa*.

The price expectations variable was subsequently implemented in a set of behavioural equations, namely the private consumption expenditure functions. Consumption expenditure was disaggregated into durable consumption, non-durable consumption and services. Since it was believed that stochastic estimation of total consumption expenditure would be more reliable than that of services, expenditure on services was deterministically determined as the residual of the total and the other two categories.

Consumption, non-human (financial) wealth and current disposable income constituted a long-run equilibrium relationship in the case of total consumption expenditure and expenditure on durables. In the case of non-durable consumption, a long-run cointegration equation included only current disposable income as an explanatory variable. Variables that contributed towards explaining the short-run dynamics of the system included wealth stock, the return on wealth, current disposable income, interest rates, relative prices and a variable reflecting labour market conditions, namely the employment rate in the non-agricultural sector. Interest rates proved to be significant in the explanation of durable consumption only, while the employment rate variable was only included in the non-durable consumption function. Apart from the above, the one-period-ahead price expectations variable (the result from the Kalman filter estimation) was included in the behavioural functions to test for the role of forward-looking inflation in consumption expenditure decisions. This variable only proved significant in the durable and total private consumption expenditure functions.

Diagnostic testing proved that the behavioural equations were statistically well specified while deterministic analysis of the response characteristics of the models proved that short and long-run response characteristics corresponded to theoretical priors and long-run equilibrium properties of the data.

From the results reported in this chapter, it may be concluded that both the time-varying Kalman filter prediction for price expectations and the estimation of the error correction models for private consumption expenditure yielded satisfactory results.