# Appendix I

# Accepted and Published Paper

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# Three-dimensional *in-situ* subsurface density estimations using a seismic technique

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**Abstract** - Various geophysical methods, such as seismic refraction and gravity techniques, are intended to measure or to give an estimation of the density of a subsurface. However, these methods, at best, yield only an average value, because of the large size of the sampling volume. Depending on the degree of accuracy needed, an undisturbed sample may well be the only solution. If it is the case, test pits need to be made or large diameter holes need to be drilled. The cost of the operation will necessarily then increases in accordance with the number of samples that need to be taken at a specific site. The entirely new geophysical method proposed in this article entails measuring the *in-situ* density of the subsurface. It utilises seismic energy by recording seismic traces in three dimensions, and measuring the density vertically, rather than laterally. Although only a few experiments have been performed so far in a "single layer" situation, the results seem promising.

**Keywords:** In-situ, density, P-wave, S-wave, Seismic

**Résumé** - Plusieurs méthodes géophysiques, telles que les techniques de réfraction sismique et gravimétrique, visent à mesurer ou à évaluer la densité de la surface. Cependant, ces méthodes, aux mieux, ne rendent qu'une valeur moyenne à cause du volume considérable du matériel prélevé. D'après le degré de précision requise, il se peut bien que la seule solution soit d'utiliser un échantillon non traité. Le cas échéant, il faudra creuser des puits d'exploration ou forer de grands trous de diamètre. Le coût de l'opération montera forcément en flèche, conformément au nombre d'échantillons qui devront être prélevés à tout site particulier. La méthode de géophysique tout à fait innovatrice proposée dans cet article nécessite que les dimensions de la densité *in-situ* du sous-sol soient mesurées. Elle se sert d'énergie sismique en enregistrant des traces sismiques à trois dimensions, et en mesurant la densité de façon verticale plutôt que latérale. Même s'il n'y a qu'un nombre limité d'expériences effectuées jusqu'au présent dans un domaine se composant d'une couche unique, les résultats de celles-ci semblent prometteurs.

## INTRODUCTION

There are various methods (e.g. the Dynamic Penetrometer Test or DPT and the Cone Penetrometer Test (Baguelin *et al*., 1978 and Franki, 1995) used today that enable structural engineers and engineering geologists to gain some understanding of the density distribution of the subsurface. These methods are, however, qualitative and give only estimates of the variations of the densities. At present the only way of obtaining conclusive measurements entails using "undisturbed samples" from the geological horizons, which can be expensive and difficult to handle. In addition to the laboratory costs, test pits or large diameter holes have to be drilled. Moreover, the unavailability of rapid laboratory results further delays the process.

A few established geophysical methods are used to determine subsurface rock densities. These include:

- the seismic method (usually refraction seismics (Kearey and Brooks, 1991)),
- the gravity method (Griffiths and King, 1969, Telford, 1986),
- and the neutron source or Troxler test (Byrne *et al*., 1995).

The main disadvantage of the first two methods is that the sampling volume of the subsurface is often prohibitively large. Seismic refraction geophone spreads can be as large as 240m. This gives rise to a density measurement that has to be averaged over a large sample volume with resulting loss of resolution. The main concerns and difficulties with the Troxler test are safety (radioactive sources), moisture content of the subsurface and the relatively shallow depths to which it can be employed (30 cm–60 cm). Best employment are usually obtained inside testpits.

Owing to these reasons, it was thus necessary to develop a new geophysical methodology or method that would not only address these needs, but that would be, as an added advantage, non-invasive (Fourie and Cole, 2004). Important advantages of such a new geophysical method would be:

- The method would require fewer test pits, large diameter holes and undisturbed samples to verify the geophysics and to construe the interpretation.
- Current structures, such as road embankments and gravity dam walls, could be investigated without compromising their integrity.
- The depth of investigation is 2 m to 5 m.

## UNDERSTANDING THE PROBLEM

All geophysical methods that are currently used to measure the density of the subsurface do so in an indirect manner. The gravity method is an exception. For example, from the seismic refraction method the P-wave velocity can be measured, and this velocity allows the scientist or engineer to derive a density (Griffiths and King, 1969, Telford, 1986, Yilmaz, 1989).

This new proposed method also measures the density of the subsurface indirectly. It is also a seismic method. It attempts to measure the amount of mass that is excited by the seismic source, as well as the volume of this mass, thereby making it possible to calculate the density of the subsurface. A much smaller sampling volume is used, which in turn improves the resolution of the density measurement.

## DESCRIPTION OF THE CONCEPT

During any seismic survey, the subsurface is set into vibration by a seismic source, such as a hammer or explosives. These vibrations are then recorded using sensitive sensors, such as geophones (Telford, 1986, Yilmaz, 1989).

In order to estimate the density of the subsurface, it is essential to obtain the mass ($M_0$) and the volume ($V_0$) of the sample area (figure 1). The challenge herein entails being able to determine $M_0$ and $V_0$ from indirect sources of measurement, without physically removing samples of the subsurface soils.

The proposed method calculates the mass and the volume of the sample to be investigated through the use of seismic waves. The seismic waves are recorded by using a three-component geophone system, mounted on a bearing plate on the surface (figure 1). The seismic energy is recorded in three dimensions, utilising the P- and S-waves, where the P-wave is the vertical wave and the S-waves are the horizontal waves. Weights are added in increments of 50 kg on top of the bearing plate to change the total mass (bearing plate plus added weights plus subsurface) and hence the frequency of vibration. The seismic source is a hammer. The P-waves and the S-waves are generated using a hammer. The hammer blows are delivered just next to the bearing plate, to simulate a zero offset (Yilmaz, 1989).

In an attempt to derive a descriptive mathematical model, it is assumed that the system can be represented by a simple model; a mass attached to a spring, which is equal to the mass of the subsurface underneath the bearing plate. The other end of the spring is fixed to an edifice (figure 2). All symbols are explained in the list of symbols (Table 1).

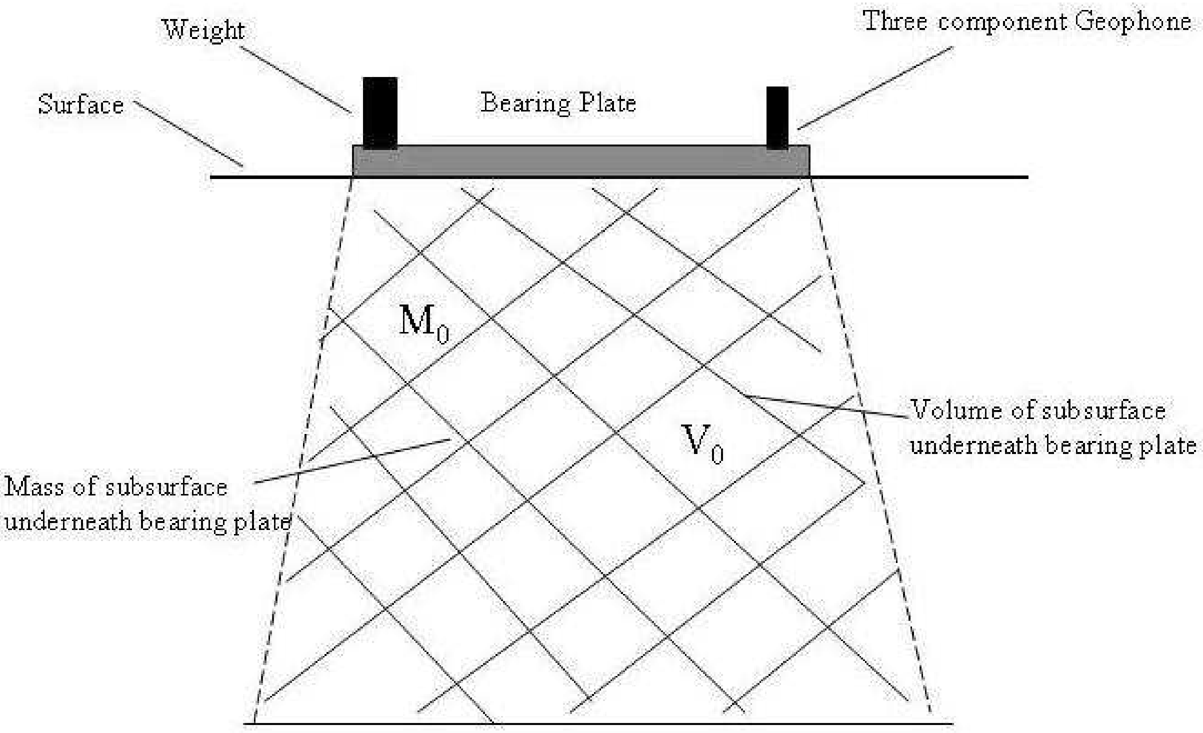

Figure 1. Schematic representation of the problem.

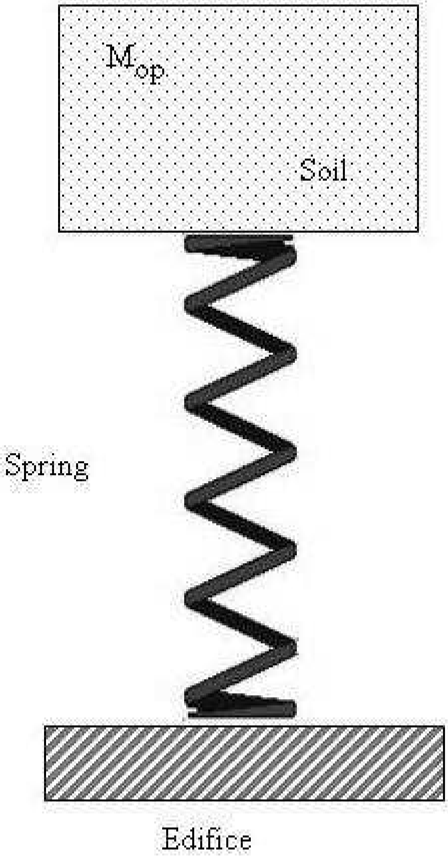

Figure 2: Simplified schematic P-wave physical model.

Table 1. List of symbols.

| SYMBOL | EXPLANATION | SYMBOL | EXPLANATION |
|---|---|---|---|
| $F_p$ | P-wave force | $M_{TZ}$ | Total mass from P-wave |
| $k_p$ | P-wave spring constant | A(t) | Amplitude with time |
| z | P-wave amplitude | $A_{0z}$ | Initial Aplitude |
| $M_{0p}$ | Groundmass | $\omega_{0z}$ | Initial Angular Frequency |
| $\ddot{z}$ | P-wave acceleration | $E_{kp}$ | Vertical Kinetic Energy |
| $\omega_p$ | Angular Frequency of the P-wave | $\rho_p$ | Vertical density |
| $\Delta m$ | Additional mass | $\lambda_p$ | P-Wave length |
| $A_p$ | Area of influence | $F_s$ | S-wave force |
| L | Length of base plate | $K_s$ | S-wave spring constant |
| $h_p$ | Depth below base plate | x | S-wave amplitude |
| $\alpha$ | Influence angle | $M_{0s}$ | Groundmass |
| $V_{0p}$ | Volume of excited mass | $\ddot{x}$ | S-wave acceleration |
| $b_p$ | Damping factor of the P-wave | $\omega_s$ | Angular Frequency of the s-wave |
| $V_p$ | Velocity of the P-wave | $M_{TX}$ | Total mass from S-wave |

### Determination of the mass $M_0$ using the P-wave

Following the above assumption, Hooke's Law (Sears *et al.*, 1987) can be written as:

$$F_p = -k_p z \qquad 1$$

where $F_p$ is the force $k_p$ is the spring constant and z is the vertical amplitude. The "p" subscript indicates the P-wave.

The differential wave equation (Kibble, 1985) that describes the movement of the mass is given in equation 2.

$$M_{0p}\ddot{z} + k_p z = 0 \qquad 2$$

By solving this equation and allowing for the addition of extra mass (Δm), the following equation follows (Fourie and Cole, 2004):

$$k_p = \omega_p^2 (M_{0p} + \Delta m) \qquad 3$$

and ω is the angular frequency.

By using equation 3 the following diagram (figure 3) represents the computation of $M_0$ (Fourie and Cole, 2004).

### Determination of the volume $V_0$ using the P-wave

If a square base plate with dimensions L is used, the resulting area underneath the plate is $L^2$. If we assume that the influence will not only come from directly underneath the plate, the new area ($A_p$) at a depth ($h_p$) below the base plate, if the angle of influence from the plate is α, is (Fourie and Cole, 2004) (figure 4)

$$A_p = (L + 2h_p \tan\alpha)^2 \qquad 4$$

where the new length is L + 2x and $\tan\alpha = x/h_p$

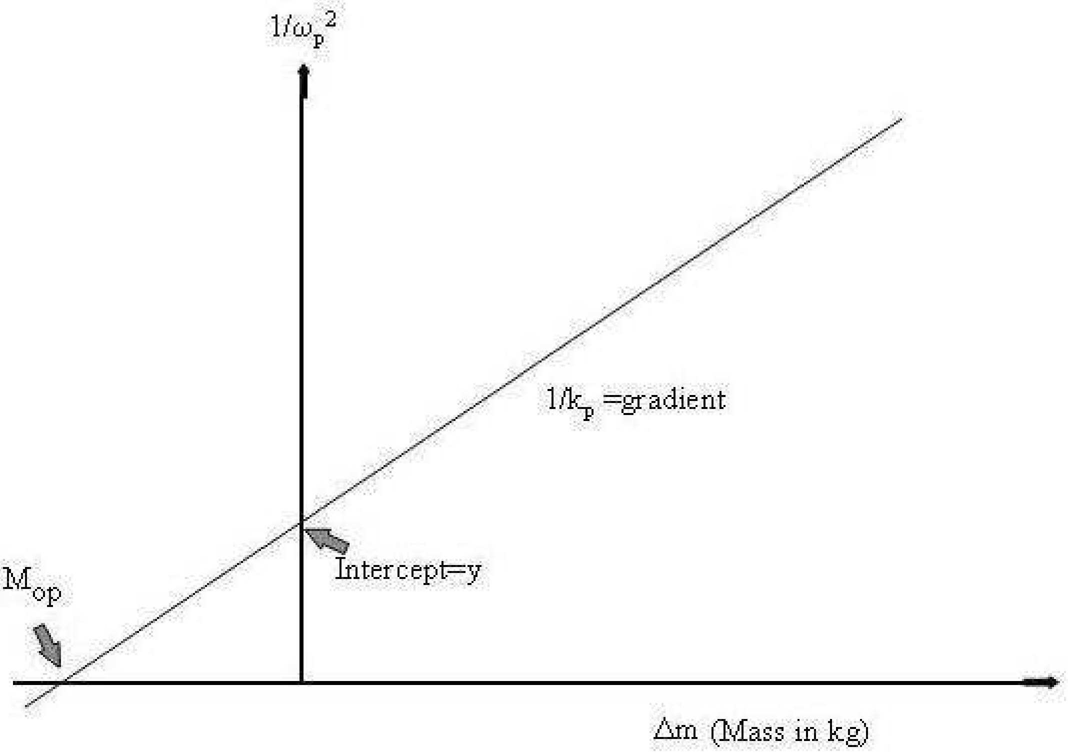

Figure 3. Schematic representation of the determination of the excited mass using the P-wave.

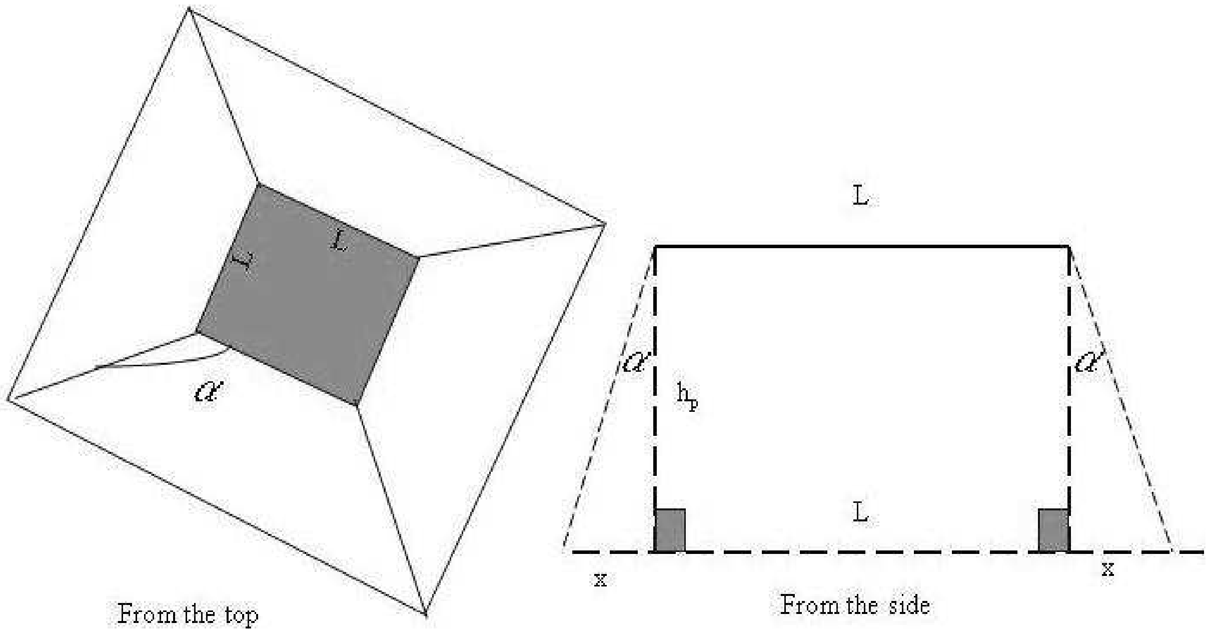

Figure 4. Determination of volume underneath the bearing plate.

The volume of the vibrating column $V_{0p} = A_p * h_p$. The height, $h_p$, of the vibrating volume as well as the angle $\alpha$, are unknown. The following deduction is based on the assumption that $\alpha$ is very small. The objective is to express the height of the volume in terms of measurable parameters, such as the wavelength or the velocity of seismic waves.

After the excitation of the groundmass, the movement is damped or attenuated. The following equation expresses the system with damping as follows (Kibble, 1985):

$$F_p + b_p V_p = -k_p z \qquad 5$$

where b is the damping factor and *V* is the velocity of the medium.

The amount of damping depends on the velocity of the movement (Kibble, 1985). If z is the movement of a differential volume under the plate, then:

$$M_{TZ}\frac{d^2 z}{dt^2} + b_p\frac{dz}{dt} + kz = 0 \qquad 6$$

where $M_{TZ}$ is the total mass.

A solution to this differential equation is (Thorne and Wallace, 1995):

$$A(t) = A_{0z}e^{-b\omega_{0z}t}\sin\omega_{0z}t\sqrt{1-b_p{}^2} \qquad 7$$

where A(t) is the amplitude with time.

If the assumption is made that the system is strongly damped, but less than critically damped, equation 7 reduces to (Thorne & Wallace 1995):

$$A(t) = A_{0z}e^{-b\omega_{0z}t} \qquad 8$$

This is a harmonic oscillation that decays exponentially with time.

If we assume that all the energy in the vertical direction (P-wave or pressure wave) is transferred from the seismic source to the ground, the kinetic energy of the source will be equal to the energy of the excited mass column. The kinetic energy ($E_{kp}$) in the vertical direction of the vibrating column is calculated, by using the following relationship (Fourie and Cole, 2004):

$$E_{kp} = \frac{1}{2}M_{0p}V_p{}^2 \qquad 9$$

The kinetic energy ($E_{kp}$) of a differential volume at a depth $h_p$, together with the facts that the mass ($M_{0p}$) = $\rho_p$*$V_p$ and the volume ($V_{0p}$) = $A_p$*$h_p$ are substituted into equation 9 results in:

$$dE_{kp} = \frac{1}{2}\rho_p A_p V_p^2 dh_p \qquad 10$$

The following equation is derived by substituting equations 4 and 8 into equation 10:

$$dE_{kp} = \frac{1}{2}\rho_p(A_0\omega_{0z})^2(L+2h_p\tan\alpha)^2 e^{-2\frac{kh}{\lambda_p}}dh_p \qquad 11$$

By solving equation 11, a mathematical relationship for the volume is derived. This volume is used with the mass to obtain the density (Fourie and Cole 2004):

$$V_{0p} = \frac{L^2\lambda_p}{2k_p} + 2L\left(\frac{\lambda_p}{k_p}\right)^2\tan\alpha + \frac{5}{2}\left(\frac{\lambda_p}{k_p}\right)^3\tan^2\alpha \qquad 12$$

The density is thus calculated by using the following relationship (Fourie and Cole, 2004):

$$\rho_{0p} = \frac{M_{0p}}{\dfrac{L^2\lambda_p}{2k_p} + 2L\left(\dfrac{\lambda_p}{k_p}\right)^2\tan\alpha + \dfrac{5}{2}\left(\dfrac{\lambda_p}{k_p}\right)^3\tan^2\alpha} \qquad 13$$

### Determination of the mass $M_0$ using the S-wave

In order to derive a mathematical model for the S-wave, the assumption is made of a mass attached to a spring at each side, to obtain a simple model. The mass attached to the springs oscillates in the S-wave (lateral) direction with host rock on both sides. Figure 5 presents this scenario schematically.

Assuming that the values of the spring constants are the same, it is then possible to write from the classical mechanics (Classical Mechanic Lecture Notes, University of Stellenbosch, 1980):

$$F_S + k_S x = -k_S x \qquad 14$$

where $k_s$ are the spring constants and x is the horizontal amplitude. The “s” subscript indicates the S-wave. All symbols are explained in the list of symbols (Table 1). The differential wave equation that describes the movement of the mass is (Classical Mechanic Lecture Notes, University of Stellenbosch, 1980):

$$M_{0S}\ddot{x} + 2k_S x = 0 \qquad 15$$

By solving equation 15, and allowing for the addition of mass, the following equation is derived (Fourie and Cole, 2004):

$$\frac{1}{\omega_S^2} = \frac{1}{2k_S}(M_{0S} + \Delta m) \qquad 16$$

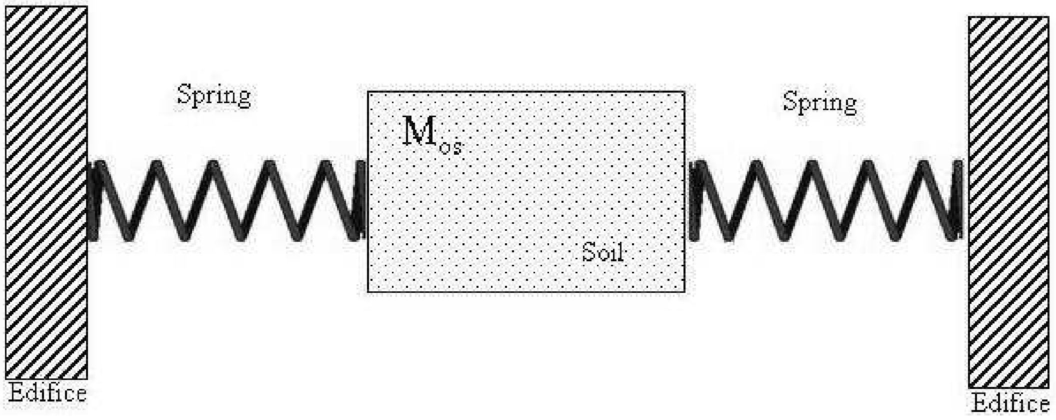

Figure 5. Simplified schematic S-wave physical model.

Equation 16 can then be used to calculate the mass that is excited by the S-wave in a similar fashion as is shown in figure 3.

### Determination of the volume $V_0$ using the S-wave

Similarly, if the same square base plate with dimensions L is used, and the angle of influence from the plate is α, the resulting area underneath the plate (figure 4) will be expressed by equation 4.

During the excitation of the groundmass, the movement is damped. The following differential equation expresses the system with damping (Classical Mechanics Lecture Notes, University of Stellenbosch, 1980):

$$F_S + b_S V_S + k_S x = -k_S x \qquad 17$$

where $b_S$ is the damping factor and $V_S$ is the S-wave velocity of the medium.

The differential wave equation describing the situation is then:

$$M_{TX}\frac{d^2x}{dt^2} + b_S\frac{dx}{dt} + 2k_S = 0 \qquad 18$$

where $M_{TX}$ is the total mass.

By going through the same process as with the P-wave, an expression for the volume of mass that is excited by the energy source can be calculated (Fourie and Cole,2004):

$$V_{0S} = \frac{L^2\lambda_s}{2k_S} + 2L\left(\frac{\lambda_s}{k_S}\right)^2 \tan\alpha + \frac{5}{2}\left(\frac{\lambda_s}{k_S}\right)^3 \tan^2\alpha \qquad 19$$

The density is thus calculated by using the following relationship (Fourie and Cole, 2004):

$$\rho_{0s} = \frac{M_{0s}}{\frac{L^2\lambda_s}{2k_s} + 2L\left(\frac{\lambda_s}{k_s}\right)^2 \tan\alpha + \frac{5}{2}\left(\frac{\lambda_s}{k_s}\right)^3 \tan^2\alpha} \qquad 20$$

## FIRST FIELD TEST ON IGNEOUS ROCKS

### Field procedure

The first density soundings were performed on an igneous complex, situated just north of Pretoria referred to in the literature as the Leeuwfontein Syenites (Wilson *et al.*, 1998) (figure 6). This site was chosen for the following reasons:

- It is a single geological layer environment.
- It was easy to drill samples from the rocks, for laboratory measurements (figure 7).

Two density soundings were performed, using a large hammer as a seismic source and a large base plate (figure 8a). P-waves (figure 8a) and S-waves (figure 8b) were generated, using the hammer. A modified base plate with steel pens is struck at the side parrallel to the surface to generate the S-waves. The seismic traces (figure 9) were recorded in three directions to obtain information in three dimensions. A laptop computer was used for the process control and the data processing. The final results are displayed in a table format.

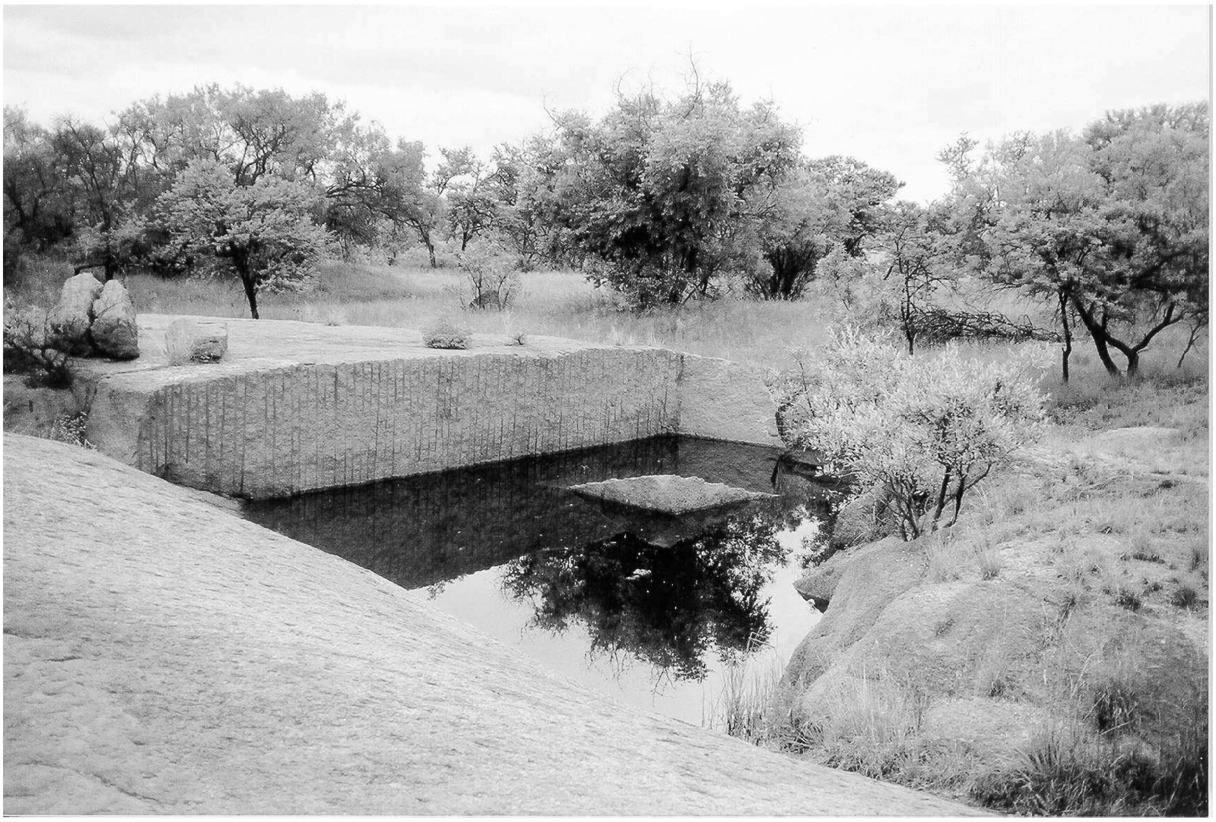

Figure 6. The Leeuwfontein Syenites just north of Pretoria. It is more than 10m thick. For the purpose of this experiment it represents a single layer.

Figure 7. Diamond core drill used for taking laboratory samples.

## RESULTS

Seismic traces and calculations were used to determine the amount of mass that had been excited by the source in three dimensions. Figure 10 shows an example of how the Z-component (P-wave) were utilised to calculate the mass..

Figure 8a. Base plate, weights and seismic hammer source to generate the P-wave.

Figure 8b: The S-wave base plate. The holes are for the steel pens. It is struck on the side.

Figure 10 shows the amount of mass that was excited to amount to 7 042 kg. A seismic P-wave velocity of 4 650 m/s was measured on the core samples in the laboratory. The attenuation factor was also calculated on the seismic traces obtained during the fieldwork. These parameters were substituted into the equation to calculate the volume, which in this case was 2,718 $m^3$. The density calculated for this sounding was 2 591 $kg/m^3$. Since the area of the base plate was 1,513 $m^2$, depth of investigation was 1,796 m. Table 2 shows a summary of the results for the two soundings performed on the Leeuwfontein Syenite.

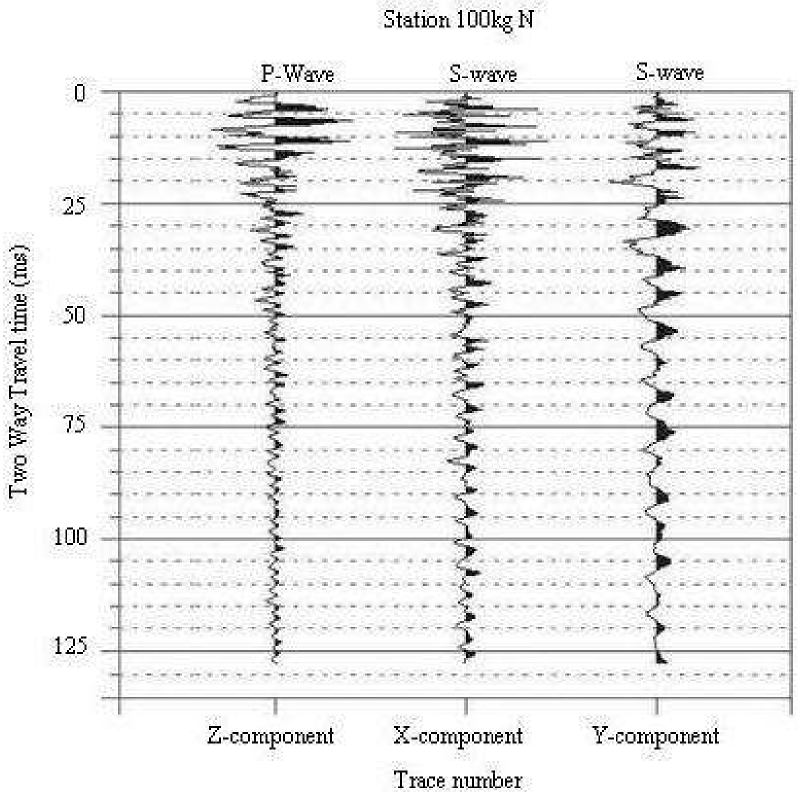

Figure 9. Seismic traces used to calculate the density.

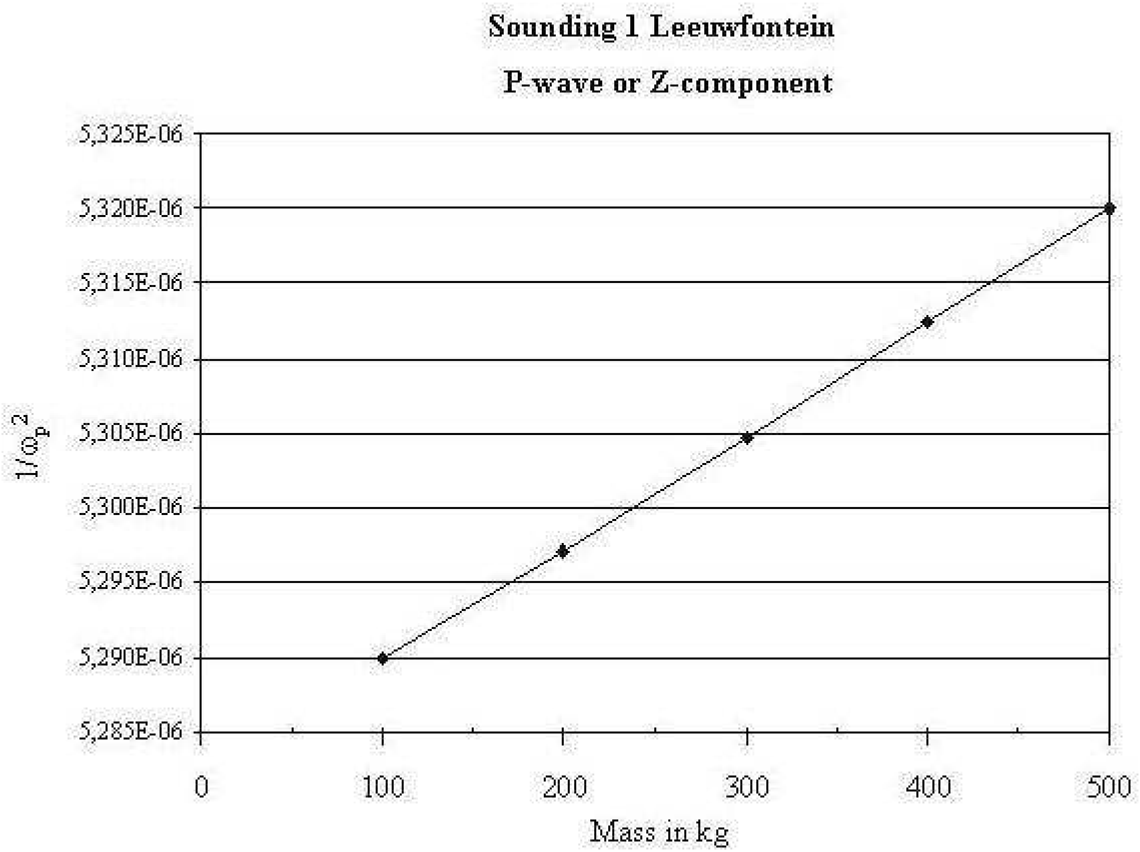

Figure 10: Plot to obtain amount of excited mass.

Table 2: Comparison between field and laboratory results

| **SOUNDING 1** | | | | | |
|---|---|---|---|---|---|
| **Mass (kg)** | **Volume ($m^3$)** | **Density (kg/$m^3$)** | **Depth (m)** | **Lab density (kg/$m^3$)** | **Difference (kg/$m^3$)** |
| 7 042 | 2,718 | 2591 | 1,80 | 2580 | +11 |
| **SOUNDING 2** | | | | | |
| **Mass (kg)** | **Volume ($m^3$)** | **Density (kg/$m^3$)** | **Depth (m)** | **Lab density (kg/$m^3$)** | **Difference (kg/$m^3$)** |
| 7 546 | 2,908 | 2594 | 1,922 | 2599 | -5 |

## DISCUSSION OF RESULTS AND THE WAY FORWARD

The development of this method was born from the need to find a way of measuring the density of the ground using a non-invasive approach. It is proposed that the new approach discussed in this paper can meet this requirement. The method uses data in three dimensions; which include the vertical pressure waves (P-waves) and horizontal shear waves (S-waves).

Comparison with the laboratory data was favourable. From Table 2 it is clear that this method was successful in the field and poses some merit, at least in a single layer situation. However, it is true that a single layer situation is encountered in very few cases. Examples of single layer cases are man-made structures, such as road embankments and gravity dam walls. It is important that the method be tested in multi-layer situations, which do not consist of igneous rock, as the method is particularly applicable to the weathered layer. In most cases, the weathered layer is a multi-layered medium.

The accuracy of the proposed method depends largely on the quality of instrumentation, processing accuracy (determining the damping factor (b)) of the seismic traces and of the subsurface. Given that the rocks and soil are strongly, but less than critically damped (Thorne & Wallace 1995) it is evident that the calculated volume depends largely on the damping factor. Various approaches to determining this factor have to be investigated on good-quality data. The coupling of the base plate with the subsurface is not perfect. This could lead to a lower determination of the excited mass and volume, and subsequently the density.

As with most of the other geophysical methods (such as the D.C. electrical resistivity and electromagnetic methods (Griffiths *et al*., 1969, Telford, 1986, Kearey *et al*., 1991), it will be impossible to do absolutely accurate interpretations without any verification. The method will yield a good first-order approximation, and if test pits or boreholes are available to verify the interpreted results, the savings may be considerable.

From the experience gained, it is evident that this method is fast. It is possible to perform about 10 soundings in one day, depending on accessibility and the terrain. A major negative factor at this stage is that the equipment is very heavy, which makes it difficult to manipulate. A redesign of the equipment will be necessary to increase the ease of handling and mobility of the equipment. One solution may be to permanently mount the equipment on a specialised vehicle.

Currently, data processing, such as the digital signal processing and filtering, are done by means of modules of various software packages. The development of dedicated software has started. This software envisages unifying all the necessary processes and routines to process the data and the display options into one package.

Finally, the main advantages of this proposed method should be emphasised:

- The method is non-invasive/non-destructive.
- The method will be cost effective, by limiting the number of test pits, and large diameter holes. It will save on laboratory costs.
- The method is fast, and the production rate is relatively high.
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## REFERENCES

Baguelin, F., Jézéquel, J. F. and Shields, D. H. 1978. *The Pressure meter and Foundation Engineering* (First Edition). Series on Rock and Soil Mechanics (Vol. 2, No. 4). Trans Tech Publications.

Byrne, G., Everett, J. P., Schwartz, K., Friedlaender, E. A., Mackintosh, N. and Wetter, C. 1995. *A Guide to Practical Geotechnical Engineering in Southern Africa*. Third Edition, Franki, 1995.

Classical Mechanics lecture notes, 1980, University of Stellenbosch, South Africa.

Fourie, C. J. S. and Cole, P. 2004. Development of an *in-situ* Density Geophysical method using a Seismic Technique. Report 2004-0095, Council for Geoscience, Pretoria, South Africa.

Fourie, C. J. S. and Cole, P. 2004. Three – dimensional *in-situ* subsurface Density estimations using a Seismic Technique. Geoscience Africa, 2004, Abstracts. (Johannesburg, South Africa).

Griffiths, D.H. and King, R.F. 1969. *Applied Geophysics for Engineers and Geologists*. Second Edition, Pergamon Press.

Kearey, Phillip and Brooks, Michael, 1991. *An introduction to Geophysical Exploration*. Second Edition, Blackwell Scientific Publications.

Kibble, T. W. B. 1985. *Classical Mechanics*. Third Edition, Longman Scientific & Technical Press, London.

Sears, Zemansky and Young, 1987. *University Physics* (Seventh Edition). Massachusetts, Addison-Wesley Publishing Company.

Telford, W. M. 1986. *Applied Geophysics*. Cambridge University Press.

Thorne, L. and Wallace, T. C. 1995. *Modern Global Seismology*. San Diego, Academic Press.

Wilson, M. G. C. and Anhaeusser, C. R. 1998. *The Mineral Resources of South Africa*. Council for Geoscience, South Africa, Sixth Edition.

Yilmaz, O. 1989. *Seismic Data Processing*, Society of Exploration Geophysicists, U.S.A.

# Appendix I

# Accepted and Revised to be Published Paper

# Three-dimensional *in-situ* small movement elasticity modulus estimations of the subsurface using a seismic technique

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**Abstract –** All civil engineering projects require information from the subsurface for design purposes. Usually this information is considered in the development of foundations of the structure. The size of the structure requires different information, mainly because foundations are smaller, shallower and located in the weathered layer for smaller structures, and deeper and larger into the base rock for larger structures. Usually collapsibility and the heaving potential play a large role in design parameters, but the small movement elasticity modulus (SMEM) is the most important constraint. Different techniques exist to measure this parameter. The most obvious one is to measure the modulus in a laboratory on an undisturbed sample. Another option recently proposed by the authors is to measure the P-wave and S-wave velocity and derive the modulus from these velocities. A new approach is to use the density sounding technique to measure the small movement elasticity modulus in three directions. In this paper we present some new experimental evidence obtained by this method.

**Keywords:** *In-situ*, density, P-wave, S-wave, small movement elasticity modulus

## La technique de sondage de densité pour mesurer le module d’élasticité de petits mouvements dans trois directions

**Résumé -** Tout projet de génie civil requiert de l’information sur la subsurface aux fins du dessin. D’habitude, cette information est considérée en vue du développement des fondations de la structure. Les dimensions de la structure nécessitent d’autres informations, surtout du fait que les fondations sont plus petites, moins profondes et situées dans la couche altérée pour des structures de dimensions moins importantes, alors qu’elles sont enfoncées plus profondément et plus largement dans la roche de base pour les structures de taille plus importante. Normalement, bien que la tendance à l’effondrement et le potentiel de la boursouflure jouent un rôle important concernant les paramètres de conception, c’est le module d’élasticité de petits mouvements (SMEM) qui y est la contrainte la plus importante. Il y a différentes techniques qui servent à mesurer ce paramètre dont le plus évident est la mesure du module dans un laboratoire en utilisant un échantillon non remanié. Une autre possibilité récemment postulée par les auteurs serait de mesurer la vitesse des ondes P et S et d’en déduire le module. Une nouvelle démarche consiste à utiliser la technique de sondage de densité pour mesurer le module d’élasticité de petits mouvements dans trois directions. Dans cet article nous visons à presenter une nouvelle évidence expérimentale obtenue par cette méthode.

**Mots clés** – *In-situ*; densité; la vitesse des ondes P et S le module d’élasticité de petits mouvements

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## INTRODUCTION

Modern developments in geophysics, geology and engineering have resulted in increasingly sophisticated techniques of analysis of problems in soil mechanics. The reasons for this are threefold:

- better and more sophisticated technology that is now more readily available.
- The need to determine parameters more accurately, to prevent large over designing.
- The need to decrease the cost of development projects.

These improved methods have also highlighted the problems inherent in conventional sampling and laboratory testing procedures. Frequently, these testing procedures cannot supply suitably accurate parameters either for sophisticated techniques of analysis (such as finite element analysis) or even for modern design calculations.

Various authors (e.g., Brown and Robertshaw, 1953; Davis and Poulos, 1966) have shown that the action of ‘sampling’ causes significant disturbance due both to mechanical deformation and to the inevitable difference in stress history between a sampled element of subsurface and a similar element in the field. This confirms the need to sample physical properties *in-situ* (for example density and small movement elasticity modulus), in particular for the construction and engineering industry.

Although tests like the Dynamic Cone Penetrometer Test and the consolidation test can give an indication of the density variations of the subsurface, accurate *in-situ* density determinations can actually only be determined up to a shallow depth by using the neutron based Troxler equipment. Variations in the moisture content can however influence the determinations. If accurate subsurface parameters were needed from greater depths, an undisturbed sample would be needed.

Physical properties derived from field geophysical techniques tend to be much greater than those obtained from conventional laboratory testing, for example stiffness derived from field seismics (Clayton and Heyman, 2001). A laser interferometry system was developed to evaluate the sensitivity and accuracy of displacement transducers (Heyman , Clayton and Reed, 1997) in order to investigate the extent of the linear-elastic range of geomaterials in triaxial stress space.

This argument has led in the past to the belief that geophysical measurements are useful in design problems associated with large events like earthquakes where large movements occur, but could not be used in engineering calculations of small ground movements around foundations and structures. This now raises the question: Is it worthwhile to develop and investigate any geophysical techniques for engineering applications?

It was argued (Auld, 1977) that seismic methods are dynamic, giving negligible time for plastic or creep strains to occur and that the induced strains are very small. It was only recently that it was appreciated that stresses and strains around tunnels are actually very small and that it follows that geophysical techniques might be capable of yielding the parameters (Jardin *et al*., 1986).

Clayton and Heyman (2001) showed that the results of the very-small-strain stiffness measured in the laboratory by using a Fabry-Perot laser interferometer under high pressure were comparable with geophysical data from the same sites where the samples were taken. The movements during the seismic experiments were measured using displacement transducers. This proves that the use of geophysical techniques is suitable in deriving *in-situ* engineering parameters and that the development of such geophysical techniques is necessary and advisable. In order to emphasize the importance of accurate *in-situ* measurements on all the physical properties that are of importance to engineers, important issues from literature should be discussed.

The Seismic Refraction method can be used to determine the densities of the subsurface (Griffiths and King, 1969; Darracott, 1976) and the small movement elasticity modulus. The densities and the small movement elasticity moduli are obtained indirectly by measuring the velocities of the P-waves and S-waves.

During a P-wave or longitudinal wave the particle movement is in the same direction as the wave propagation, in other words a pressure wave. During the S-wave or transverse wave the particle movement is perpendicular to the wave propagation. The equations describing these velocities are:

$$V_p = \sqrt{\frac{k + \frac{4}{3}G}{\rho}} \qquad 1$$

and

$$V_s = \sqrt{\frac{G}{\rho}} \qquad 2$$

where **k** is the bulk or incompressibility modulus. G is the shear or rigidity modulus, and ρ is the density.

From the above equations it is obvious that larger modulus values are associated with higher velocities. By using this method, the density and modulus are measured *in-situ*. These values are however only an indication of the real value due to the large sampling volume, which is directly proportional to the size of the seismic spread (6,12,24 channel). The sampling volume can be reduced if smaller geophone spacing is used.

Engineers routinely use the ratio of $V_p/V_s$ to give them an indication of the density and “hardness” of the subsurface, and generally the following hold true (Darracott, 1976):

- High $V_p$ and a $V_p/V_s$ ratio of approximately $\sqrt{3}$ indicates unweathered bedrock.
- Low $V_p$ and a $V_p/V_s$ ratio of approximately $\sqrt{3}$ indicates sandy or gravel fill.
- Low $V_p$ and a high $V_p/V_s$ ratio indicate clayey material, usually above the water table.
- $V_p$ velocity about 1500 m/s, high $V_p/V_s$ ratio may indicate soft clay material, below the water table.

It is impossible to distinguish between minor layers inside the weathering layer if the velocity contrasts are small. For most rocks there is an empirical relationship between the $V_p$ and the rock quality, namely the higher the velocity the better the rock quality (Brown and Shaw, 1953). They showed the empirical relationship between Young’s modulus (E) and $V_p$:

$$E = 111.15 V_P^{\,2.34} \qquad 3$$

where the unit for E is in Pa.

Poisson ratios can be obtained from seismic velocities. It is the ratio between compression strain and extension strain in the direction of stretching force. Extensive (stretching) deformation is considered positive and compressive deformation is considered negative. The definition of Poisson's ratio contains a minus sign so that normal materials have a positive ratio.

$$\nu = \frac{-\varepsilon_{compressive}}{\varepsilon_{extensive}} \qquad 4$$

where

$$\frac{V_p}{V_s} = \sqrt{\frac{1-\nu}{\frac{1}{2}-\nu}} \qquad 5$$

If the subsurface is compacted, it is found that the elastic modulus K and G increase more rapidly than does the density, which makes the determination of the density difficult (Griffiths and King, 1969).

Since it is difficult to assign a specific velocity and a density to a rock type, it is however possible to quote a range of velocities which would cover a certain lithology. Table 1 shows the general trend of increasing velocity with increasing density (i. e., decreasing porosity). The large areas of overlap indicate the insensitivity of the seismic velocity to small variations in density.

Materials of exceptionally low velocity are usually encountered near the surface (weathered layer) and are of considerable importance to the civil engineer. Properties such as its elasticity (especially the small movement elasticity modulus), plasticity, strength and density are the most important. Young’s modulus can only be determined if the velocity, density and Poisson’s ratio are known.

The seismic cone test (Heyman, 2003) is used to measure the *in situ* S and P waves of the soil. The largest advantage of this test is that it allows for the measurement of the void ratio on undisturbed material at the *in situ* stress condition. Heyman (2003) used the seismic cone test on a gold tailings dam and compared it with an undisturbed sample from the daywall of the same dam. According to Heyman (2003) the small strain stiffness and Poisson’s ratio can also be calculated from the velocity measurements where small movements occur, such as foundations.

Shear waves provide a direct way of determining the dynamic shear modulus of the ground that is independent of Poisson’s ratio (Abbiss, 1981). Two shear wave methods have been applied to determine *in situ* properties as a function of depth on three clay sites. The first method was shear wave refraction and the second measured the velocity of Rayleigh waves

**Table1**. P-wave velocities for certain rock types (after Griffith and King, 1969).

*Tableau 1 . in French*

| **Lithology** | **P-wave velocity (km/s)** | **Density (g/cm$^3$)** |
|---|---|---|
| Clastic rocks, unconsolidated | 0.3-1.8 | 1.5-2.2 |
| Clastic rocks, consolidated and cemented | 1.5-3.7 | 2.0-2.6 |
| Clastic rocks in orogenic belts | 3.1-6.2 | 2.5-2.8 |
| Metamorphic rocks | 4.6-6.2 | 2.7-3.0 |
| Limestone | 3.1-6.2 | 2.4-2.7 |
| Igneous rocks | 4.6-4.2 | 2.4-3.0 |

generated by vibrators. In addition pressure wave velocities were measured enabling the dynamic Poisson's ratio to be calculated.

Shear waves methods have the advantage that the shear modulus is directly related to the shear wave velocity (Equation 2, Abbiss, 1981). This is not necessarily the case for the p-wave velocity where the modulus may depend to a large extent on Poisson's ratio.

The modulus of chalk at Munford was compared from a seismic survey and a large scale tank test (Abbiss, 1979). A steel tank of 18.3m in diameter and approximately 20m high was filled with water to produce a pressure of 179kNm$^{-2}$. Displacements under the tank were measured in vertical shafts, by means of very accurate displacement transducers. In this way strains were measured at various levels down to 16.3m below the tank, nearly to the water table. It was found that the dynamic Young's moduli calculated from a seismic refraction survey of the chalk are proportional to the moduli determined from the full scale tank loading test. The moduli showed an approximately linear increase with depth similar to a modified Gibson soil of the first kind.

The main problem of a shear wave refraction survey is to identify the s-wave arrivals between the p-waves. The best way to remedy the situation is to use a source that is rich in s-waves in the direction of the survey (Abbiss, 1981). The signal to noise ratio can also be improved by stacking the signal on a seismograph. By reversing the connections of the s-wave geophone will help stacking of the s-wave but zero the p-wave (Abbiss, 1981).

The seismic sounding technique (Fourie, 2005, Fourie and Cole, 2004b) uses seismic waves in three directions to obtain the moduli in three directions. A large base plate is used and the mass on top of the base plate is increased inclemently. From this graph the moduli are calculated, without the necessity to obtain seismic velocities.

## UNDERSTANDING THE PROBLEM

All geophysical methods that are currently used to measure the moduli of the subsurface do so in an indirect manner. From the seismic refraction method the P-wave velocity can be measured, and this velocity allows the scientist or engineer to derive a modulus if the density is known (Griffiths and King, 1969; Telford, 1986; Yilmaz, 1989). If an S-wave velocity is known, the process is even easier.

This new proposed method also measures the modulus of the subsurface indirectly. It is also a seismic method (Fourie, 2005; Fourie and Cole, 2004a). It attempts to measure the amount of mass that is excited by the seismic source. By adding weights to a base plate, the vibration frequency of the system is changed, thereby making it possible to calculate the elasticity modulus of the subsurface. A much smaller sampling volume is used, which in turn improves the accuracy of the measurement.

In order to estimate the small movement elasticity modulus of the subsurface, the same process as determining the density of the subsurface is followed. During the process of obtaining the mass ($M_0$) of the density sounding technique (Fourie, 2005) the small movement elasticity modulus is acquired.

## DESCRIPTION OF THE CONCEPT

The P- and S- seismic waves are recorded by using a three-component geophone system, mounted on a bearing plate on the surface (Figure 1). Weights are added in increments of 50 kg on top of the base plate to change the total mass (base plate plus added weights plus subsurface) and hence the frequency of vibration (Fourie, 2005). For a complete description of the method see Fourie, 2005.

The equation that describes the calculation of the modulus versus the addition of extra mass (Δm) after Fourie (2005) is:

$$k_p = \omega_p^2 (M_{0p} + \Delta m) \qquad 6$$

where ω is the angular frequency.

By using equation 6, the computation of the bulk modulus $k_p$ (Fourie, 2005) is achieved (Figure 2). All symbols are explained in the list of symbols (Table 2).

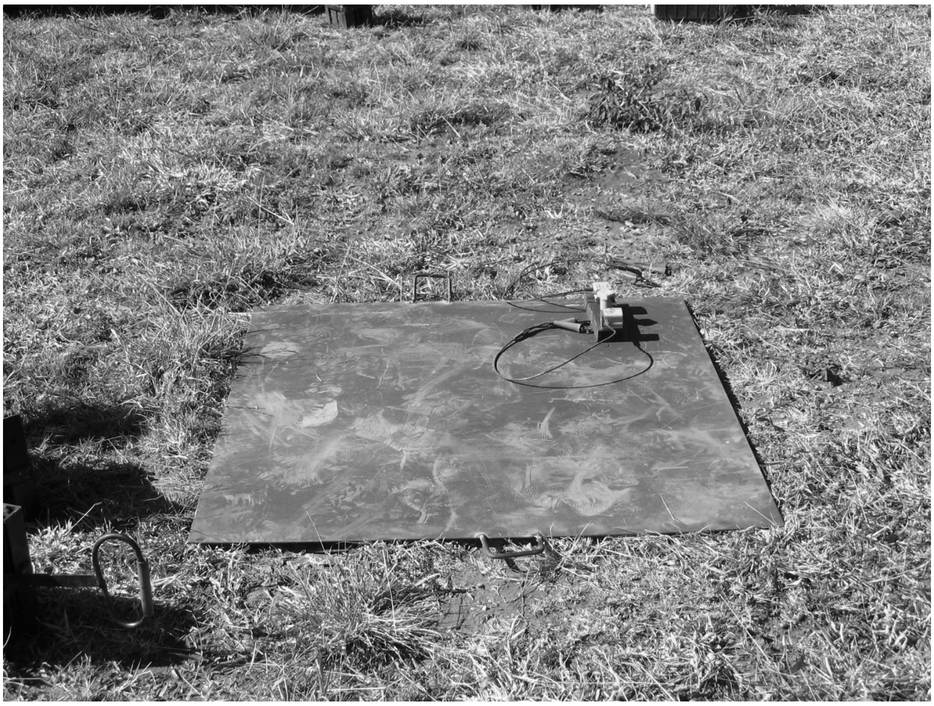

**Fig. 1**. Large bearing plate with geophones mounted for measurements in three directions.

*Fig. 1 . in French*

**Table 2**: List of symbols.

*Tableau 2. in French*

| **SYMBOL** | **EXPLANATION** | **SYMBOL** | **EXPLANATION** |
|---|---|---|---|
| $F_p$ | P-wave force | $M_{TZ}$ | Total mass from P-wave |
| $k_p$ | P-wave spring constant | $M_{TX}$ | Total mass from S-wave |
| z | P-wave amplitude | $\omega_{0z}$ | Initial Angular Frequency |
| $M_{0p}$ | Groundmass | $\omega_s$ | Angular Frequency of the s-wave |
| $\ddot{z}$ | P-wave acceleration | $F_s$ | S-wave force |
| $\omega_p$ | Angular Frequency of the P-wave | $K_s$ | S-wave spring constant |
| $\Delta m$ | Additional mass | x | S-wave amplitude |
| $\ddot{x}$ | S-wave acceleration | $M_{0s}$ | Groundmass |

In a similar fashion, the equation that describes the calculation of the shear modulus ($k_s$) versus the addition of extra mass ($\Delta$m) after Fourie (2005) is:

$$\frac{1}{\omega_S^2} = \frac{1}{2k_S}(M_{0S} + \Delta m) \qquad 7$$

Equation 7 can then be used to calculate the shear modulus ($k_s$) versus the addition of extra mass ($\Delta$m) in a similar fashion and is shown in Figure 2.

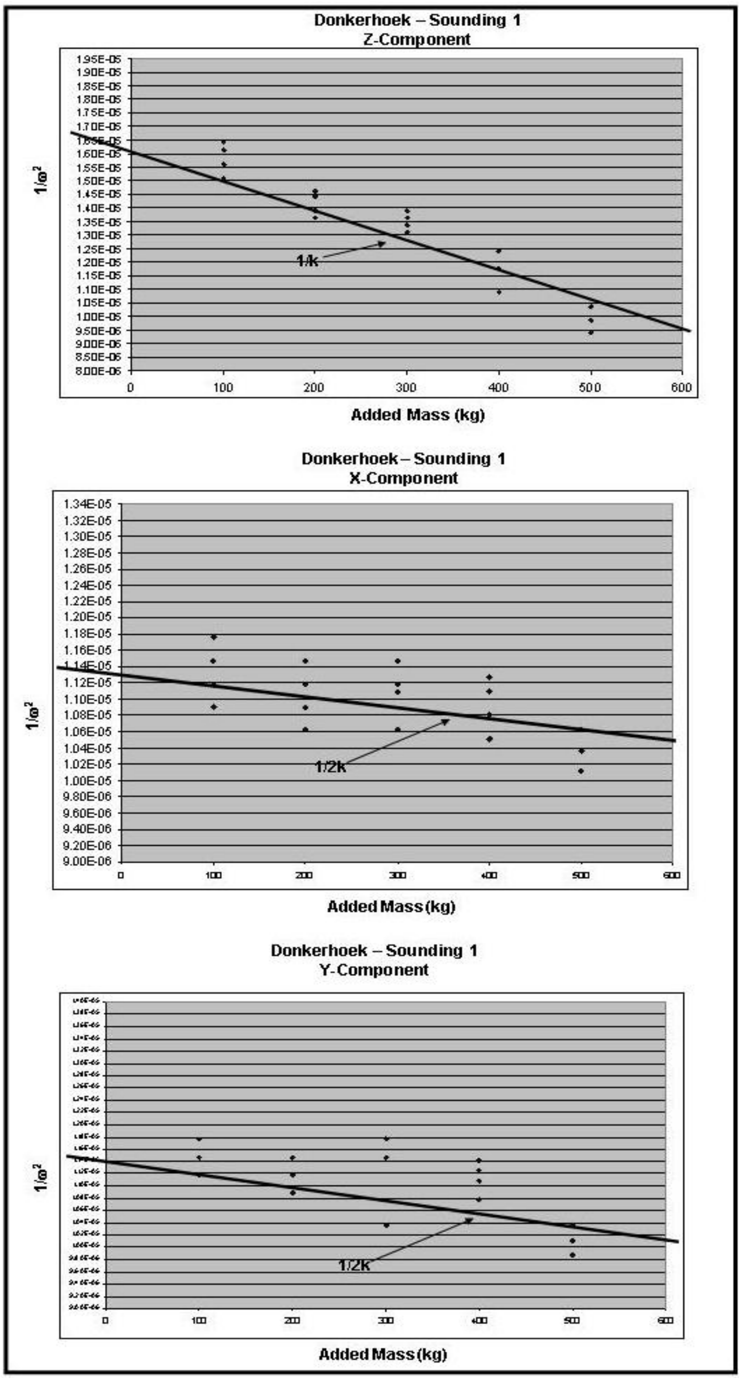

**Fig. 2**. Example of calculation of the excited mass and the modulus for each direction. Note that modulus is 1/k for the P-wave and 1/2k for the S-wave.

*Fig. 2. in French*

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# FIELD TEST ON WEATHERED MIDRAND GRANITES

## Field procedure

The density soundings were performed on the weathered layer of an igneous complex, situated just south of Pretoria referred to in the literature as the Midrand Granites (Wilson *et al*., 1998) (Fig. 3).

These granites are Archean basement and weathers to an *in-situ* clay (Fig. 4). The purpose of the survey was to compare this geophysical method with laboratory results for this area as part of an urban development.

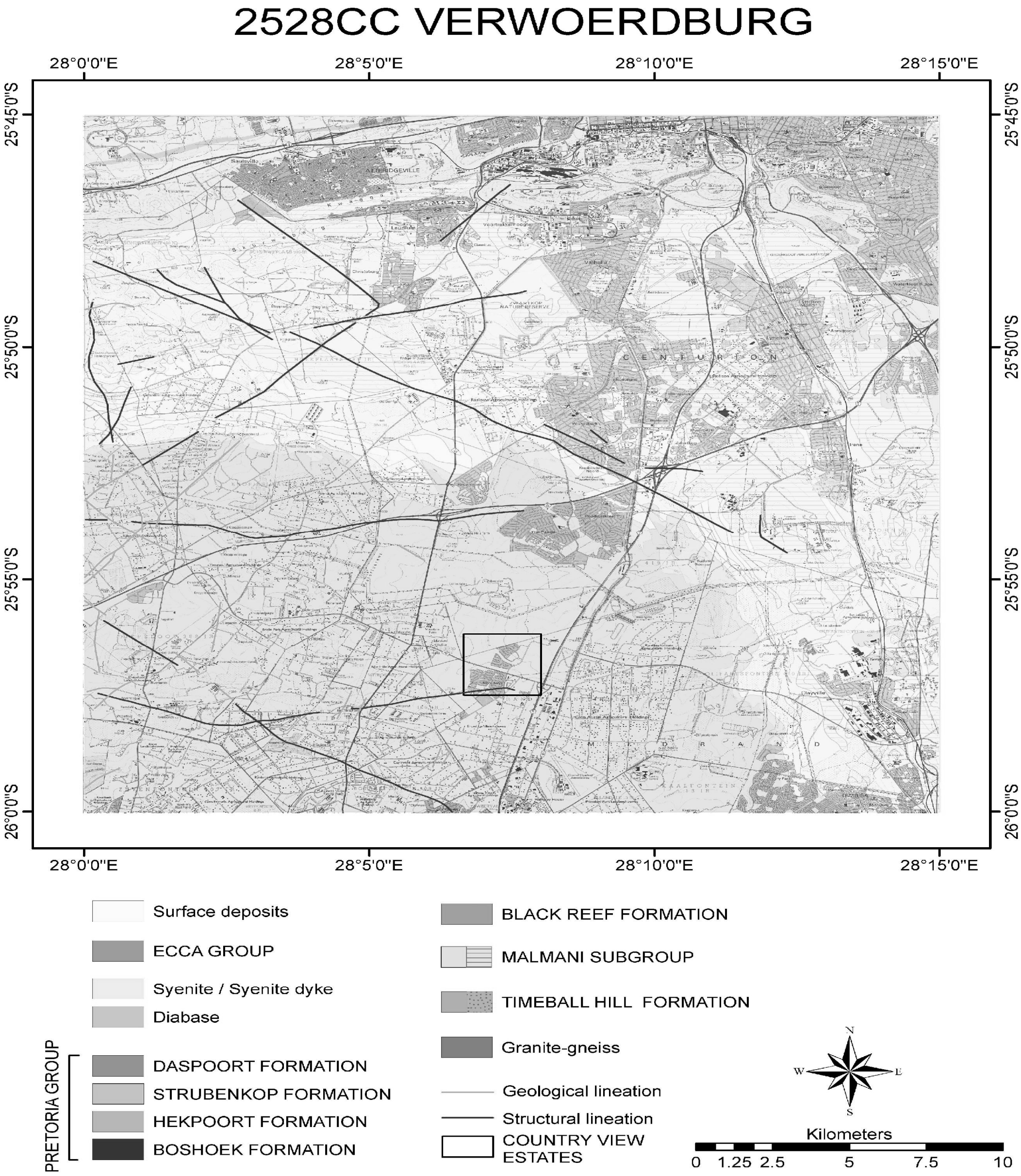


**Fig. 3**. 1:50 000 sheet indicating the survey area inside the rectangle.

*Fig.3. in French*

**Fig. 4**. Weathering profile of the Midrand Granites. From this figure the different layers can be seen.

*Fig. 4. in French*

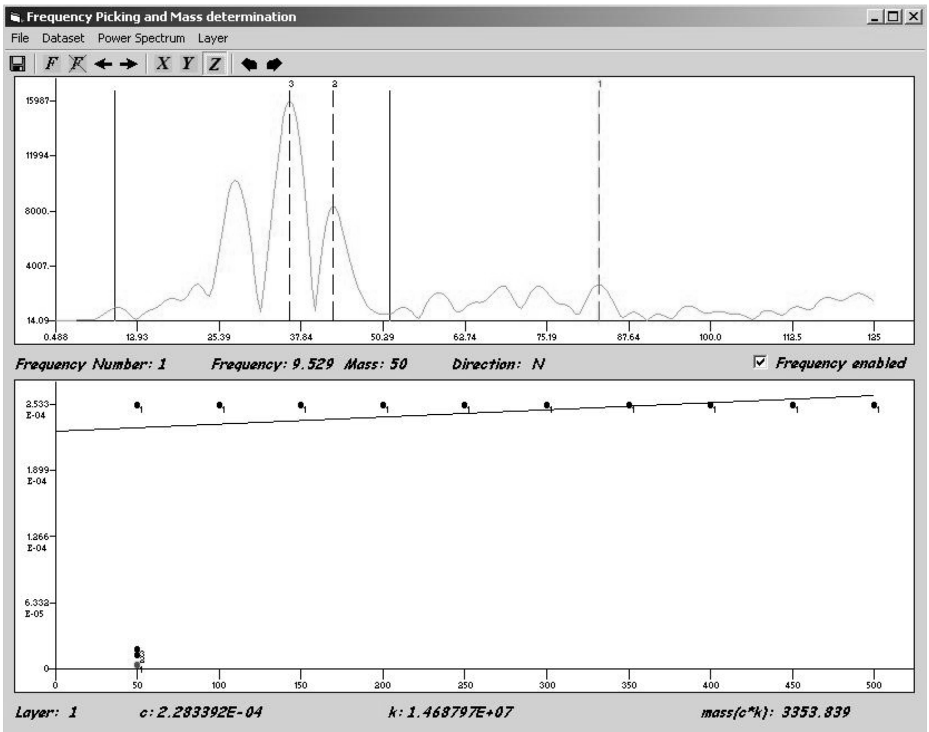
Five density soundings were performed, using a large hammer as a seismic source and a large base plate (Fig. 1). P-waves and S-waves were generated, using the hammer. The seismic traces were recorded in three directions to obtain information in three dimensions. A seismograph was used to record the data. The data were processed using an in-house developed software package called Seisrho.

The seismic record data from the seismograph is read into Seisrho. A Fourier transform is then calculated on each trace to obtain a frequency spectrum. This frequency spectrum is then used to pick the frequencies that correspond to each applied mass and layer. The process is repeated till a straight line can be constructed to obtain the modulus values and the excited masses. All this processing is done in the window of Seisrho that is displayed in Figure 5.

## RESULTS

Figure 5 shows the window in Seisrho where the excited masses are calculated for each layer. Each line represents a different layer. The modulus values are also estimated inside this window of Seisrho, by calculating the gradients of each line. This is achieved by calculating the frequency response for each trace at different applied masses on the base plate. An example of the final results is displayed in Table 3. A comparison of the field data to the laboratory results of Soillab is also given in Table 3. The modulus values were obtained in the laboratory on undisturbed samples by using the double odometer test.

Seismic traces and the calculations in Seisrho were used to determine the amount of mass that had been excited by the source in three dimensions and also the modulus values. The results indicate that the second layer of the area is weaker in the E-W direction than the first and third layers. The results thus indicate the anisotropic nature of the site, meaning that the structural support of the foundations by the weathered layer will be larger in the N-S direction as in the E-W direction. Otherwise the foundations must be deeper than the second layer to avoid possible sagging problems. This information indicates towards the civil engineer that the design should incorporate this by rotating the structures in such a way that the largest strains are in the stronger E-W direction, or that special instructions are given for the construction.



Frequency Number: 1 Frequency: 9.529 Mass: 50 Direction: N ☑ Frequency enabled

Layer: 1 c: 2.283392E-04 k: 1.468797E+07 mass(c*k): 3353.839

Fig. 5. Processing window in Seisrho where the modulus values are calculated.

Table 3. Comparison of field data with laboratory data.

| P-Wave | | | | |
|---|---|---|---|---|
| **Layer** | **Total Mass** | **Mass** | **E-Modulus** | **E-Mod(lab)** |
| 1 | 1309 | 1309 | 367MPa | 400MPa |
| 2 | 4991 | 3683 | 373MPa | |
| 3 | 6439 | 1448 | 374MPa | |
| **S-Wave (N-S Direction)** | | | | |
| **Layer** | **Total Mass** | **Mass** | **E-Modulus** | **E-Mod(lab)** |
| 1 | 1318 | 1318 | 338MPa | |
| 2 | 4966 | 3648 | 778MPa | 800MPa |
| 3 | 6427 | 1461 | 789MPa | |
| **S-Wave (E-W Direction)** | | | | |
| **Layer** | **Total Mass** | **Mass** | **E-Modulus** | **E-Mod(lab)** |
| 1 | 1313 | 1313 | 338MPa | |
| 2 | 4992 | 3679 | 105MPa | |
| 3 | 6496 | 1504 | 432MPa | |
| | | **Ave1** | **328MPa** | |
| | | **Ave2** | **434MPa** | |
| | | **Ave3** | **532MPa** | |

## CONCLUSIONS

The development of this method was born originally from the need to find a way of measuring the density of the ground using a non-invasive approach (Fourie, 2005). Other than the seismic refraction method, the determination of the modulus, without having a seismic velocity, is an extra benefit from the method. Four different surveys should be done if the seismic refraction method is to be used to obtain data in three directions. This method acquires data in three dimensions at the same time, saving a lot of time. Information about the anisotropy of the subsurface is also recorded at the same time.

Comparison with the laboratory results was favourable, as can be seen from Table 3. The accuracy of this method depends largely on the quality of instrumentation, processing accuracy of the seismic traces and of the subsurface. The coupling of the base plate with the subsurface is not perfect. This could lead to a lower determination of the excited mass and subsequently the modulus. It will be impossible to do absolute accurate interpretations without any verification. If test pits or boreholes are available to verify the interpreted results, the financial and time savings may be considerable.

Finally, the main advantage of this proposed method is that it is an alternative to the seismic refraction and the seismic cone test methods. Triggering problems can introduce errors in seismic velocities, which in turn can give faulty modulus values. The largest advantage of this test is that no knowledge of the seismic velocity is needed.

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## REFERENCES

Abbiss, C. P. 1979. A comparison of the stiffness of the chalk at Mundford from a seismic survey and a large scale tank test. *Geotechnique* **29**, no. 4, 461-468.

Abbiss, C. P. 1981: Shear wave measurements of the elasticity of the ground. *Geotechnique* 31, no.1, 91-104.

Auld, B. 1977: Cross - hole and down - hole $V_s$ by mechanical impulse. *Journal Geotechnical Engineering* **103**, no. GT12,. 1381-1398.

Brown, P. D. and Robertshaw, J. 1953. The *in-situ* measurement of Young's modulus for rock by a dynamic method. *Geotechnique* **3**, no. 7, 283.

Davis, E. H. and Poulos H. G. 1966: Laboratory investigations of the effects of sampling; *Proceedings. Symposium on Site Investigation*, Sydney, Australia.

Clayton, C. R. I. and Heyman, G. 2001. Stiffness of geomaterials at very small strains. *Geotechnique* **51**, no. 3, 245-255.

Darracott, B. W. 1976. Seismic surveys and civil engineering. *The Civil Engineer in South Africa*, February, 1976.

Fourie, C. J. S. 2005. Three-dimensional *in-situ* subsurface density estimations using a seismic technique. *Africa Geoscience Review* **12**,.nos 1 & 2, 1-12.

Fourie, C. J. S. and Cole, P. 2004a. Development of an *In-situ* Density Geophysical method using a Seismic Technique. Report 2004-0095, Council for Geoscience, Pretoria, South Africa.

Fourie, C. J. S. and Cole, P. 2004b. Three–dimensional *In-situ* subsurface Density estimations using a Seismic Technique. Geoscience Africa, 2004, *Abstracts*. (Johannesburg, South Africa).

Griffiths, D. H. and King, R. F. 1969. *Applied Geophysics for Engineers and Geologists*. Second Edition, Pergamon Press.

Heyman, G. 2003. The Seismic Cone Test; *Journal South African Institution Civil Engineering* **45**, no.2, p.26-31, Paper 552.

Heyman, G., Clayton, C. R. I. and Reed, T. 1997. Laser Interferometry to evaluate the performance of local displacement transducers. *Geotechnique* **47**, no. 3, 399-405.

Jardin, R. J., Potts, D. M., Fourie, A. B. and Burland, J. B. 1986. Studies of the influence of non-linear characteristics in soil-structure interaction. *Geotechnique* **36**, no. 3, 377-396.

Telford, W. M. 1986. *Applied Geophysics*. Cambridge University Press, U. K..

Wilson, M. G. C. and Anhaeusser, C. R. 1998. *The Mineral Resources of South Africa*. Council for Geoscience, South Africa, Sixth Edition.

Yilmaz, O. 1989. *Seismic Data Processing*, Society of Exploration Geophysicists, U.S.A.