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Analysis and Optimum Design of Stiffened Shear Webs in Airframes

by

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Figure 1: An illustrative example of shear buckling (Reproduced from *Flight International*, 8-14 September, 1999)

Abstract

Title:	Analysis and Optimum Design of Stiffened Shear Webs in Airframes
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The analysis and optimum design of stiffened, shear webs in aircraft structures is addressed. The post-buckling behavior of the webs is assessed using the iterative algorithm developed by Grisham. This method requires only linear finite element analyses, while convergence is typically achieved in as few as five iterations. The Grisham algorithm is extensively compared with empirical analysis methods previously used for aircraft structures and also with a refined, non-linear quasi-static finite element analysis.

The Grisham algorithm provides for both compressive buckling in two directions as well as shear buckling, and overcomes some of the conservatism inherent in conventional methods of analysis. In addition, the method is notably less expensive than a complete non-linear finite element analysis, even though global collapse cannot be predicted. While verification of the analysis methodology is the main focus of the study, an initial investigation into optimization is also made. In optimizing stiffened thin walled structures, the Grisham algorithm is combined with a genetic algorithm. Allowable stress constraints are accommodated using a simple penalty formulation.

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Opsomming

Titel:	Analise en Optimale Ontwerp van Verstyfde Skuifpanele in Lugrame
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Sleutelwoorde:	Post-knik analise, Grisham algoritme, diagonale trekspanning, NACA, nie-lineêre eindige element analise, skuifpaneel, vliegtuig strukture, FORTRAN program, struktuur optimering, genetiese algoritme

Die analise en optimale ontwerp van verstyfde, dunwand skuifpanele in lugvaartstrukture word aangespreek. Die post-knik gedrag van die panele word evalueer met 'n iteratiewe algoritme wat deur Grisham ontwikkel is. Hierdie metode gebruik slegs lineêre eindige element analises en konvergeer gewoonlik binne so min soos vyf iterasies. Grisham se algoritme word vergelyk met empiriese metodes wat voorheen in lugvaartstruktuuranalise gebruik is, asook met 'n nie-lineêre eindige element analise.

Daar word getoon dat die Grisham algoritme voorsiening maak vir samedrukkingsknik in twee rigtings asook knik in skuif, en die algoritme oorkom die konserwatiewe benadering wat in konvensionele ontwerpsmetodes gebruik word. Boonop is die metode ook nie so duur soos nie-lineêre eindige element analises nie, alhoewel dit nie globaal die swigting van 'n struktuur kan voorspel nie. Alhoewel verifikasie van die analiseringsmetodologie die hoof doel van hierdie studie is, word 'n aanvanklike ondersoek in optimering ook gedoen. Om die verstyfde, dunwand strukture te optimeer word die Grisham algoritme gekombineer met 'n genetiese algoritme. Aanvaarbare spanningswaardes word in ag geneem deur 'n eenvoudige boetefunksie te gebruik.

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List of symbols

A_u - Upright area
A_f - Flange cross-sectional area
A_{f_u} - Upper flange cross-sectional area
A_{f_l} - Lower flange cross-sectional area
c - Damping factor
d - Spacing of uprights
E_f - Young's modulus of flange
E_u - Young's modulus of upright
E_w - Young's modulus of web
E_{w_x}, E_{w_y} - Post-buckled Young's modulus of web
G_w - Shear modulus of web
G_{pdt} - Shear modulus of web associated with pure diagonal tension
G_{IDT} - Shear modulus of web associated with incomplete diagonal tension
h - Depth of beam
h_c - Clear depth of web, either measured between the inside tips of the
- two flange cross-sections or between the rivet rows of the upper
- and lower flanges
h_e - Effective depth of beam measured between the centroids of the two
- flanges
k - Diagonal tension factor
l - Total length of cantilever beam
L_x - Length of web along the x-axis
L_y - Length of web along the y-axis
N - Internal loads of structure
P - Applied load
S_b - Applied shear load at the onset of buckling
t - Thickness of web
t_u - Thickness of upright
u_1, u_2, u_3 - Displacement in global x-, y- and z-directions

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α - Diagonal tension angle
α_{prin} - Angle of major principal stress
ϵ_x, ϵ_y - Web normal strain
ϵ_{x_c}, ϵ_{y_c} - Web compressive buckling strain
$\epsilon_{x_{DT}}$, $\epsilon_{y_{DT}}$ - Web diagonal tension strain
σ_1 - Maximum principal stress
σ_2 - Minimum principal stress
σ_{diag} - Web diagonal tension stress along the diagonal tension angle
$\sigma_{f_{cpl}}$ - Crippling stress of flange
σ_{f_l} - Lower flange stress
σ_{f_u} - Upper flange stress
σ_{mises} - Von Mises stress
σ_{norm} - Web stress perpendicular to the diagonal tension stress
σ_u - Upright stress
$\overline{\sigma_u}$ - Average upright stress
σ_{ut} - Ultimate tensile strength in tension
$\sigma_{u_{max}}$ - Maximum stress in the upright
σ_x, σ_y - Web normal stress
σ_{x_c}, σ_{y_c} - Web compressive buckling stress
$\sigma_{x_{cr}}$, $\sigma_{y_{cr}}$ - Web modified critical buckling normal stress
$\sigma_{x_{cr_0}}$, $\sigma_{y_{cr_0}}$ - Web critical buckling normal stress based on geometry only
$\sigma_{x_{DT}}$, $\sigma_{y_{DT}}$ - Web diagonal tension stress
σ_{yt} - Yield strength in tension
τ_{cr} - Web critical buckling shear stress
τ_{DT} - Diagonal tension component of shear stress carried by web
τ_s - Pure shear component of shear stress carried by web
τ_{xy} - Web shear stress
$\tau_{xy_{cr}}$ - Web modified critical buckling shear stress
$\tau_{xy_{cr_0}}$ - Web critical buckling shear stress based on geometry only
γ_{xy} - Web shear strain
γ_{xy_c} - Web shear strain component due to compressive buckling
$\gamma_{xy_{DT}}$ - Post-buckled shear strain component of web
μ - Poisson's ratio

Subscripts 'x' and 'y' indicate the global axes.