Chapter 8

Comparison Between Theory and Practice

8.1 Background

Theoretical analysis results are discussed in conjunction with the measured values to evaluate the use of the models for SFRC and further compare the two slabs.

8.2 Results

8.2.1 Westergaard K-value

Figure 8-1 shows the stress-deflection relation for the bearing-plate test conducted on the foamed concrete subbase.

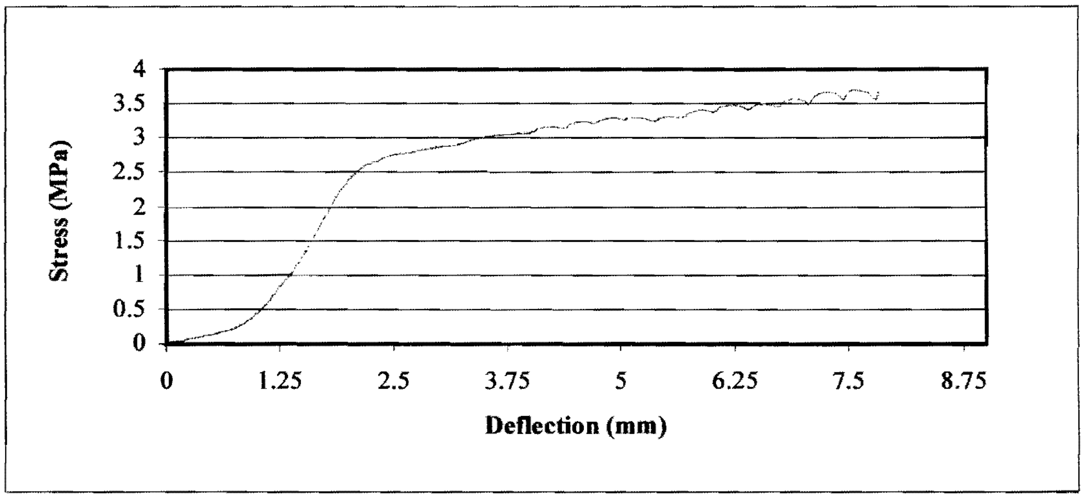

Figure 8-1: Stress-Deflection Diagram: Plate-Bearing Test on Foamed Concrete Sub base.

Based on figure 8-1, the stress at 1.25 mm deflection is 0.8 MPa. Thus K-value relative to 250 mm plate is 0.8/1.25, which equal 0.64 MPa/mm. The correction factor is used to rectify for the plate size and the K-value as given by Westergaard is (0.64/2.55) that is *0.25 MPa/mm.*

8.2.2 Characters Used for Analysis

Table 8-1 shows the concrete properties as required by Westergaard, Meyerhof, Falkner et al and Shentu et al models to assess the load capacity and /or the vertical deflection at loading points.

Table 8-1: Properties of the Slabs Mix

Property	Units	P.C. Slab	SFRC Slab	Remark
Nominal thickness	mm	150	125	Casting depths
First crack strength (f_{ct})	MPa	4.1	4.8	Measured from Third-Point loading test
After crack strength ($f_{e,3}$)	MPa		1.9	Calculated from third-point loading test data.
Equivalent strength ratio ($R_{e,3}$)	%		39.6	Calculated from third-point loading test data.
E-value	MPa	24000	27500	Calculated from a third-point loading test.
Poisson's ratio		0.15	0.15	Estimated (has very little influence)
Uniaxial tensile strength.	MPa	2.2	2.2	Estimated
K-value (28 days)	MPa/mm	0.25	0.25	Measured from bearing plate test
(90 days)	MPa/mm	0.3	0.3	Estimated

8.2.3 Interior Load Capacity

Table 8-2 shows the results of calculation of interior load capacity of the two slabs by using Westergaard, Meyerhof, Falkner et al and Shentu et al. The calculation was performed using the properties as indicated in table 8-1. Apart from the measured K-value, various other K-values were assumed and the relevant load capacities were calculated.

Table 8-2: Calculated Interior Load Capacity for Various K-values

K	Plain Concrete Slab (150 mm thickness) (KN)				SFRC Slab (125 mm thickness) (KN)			
(N/mm^3)	Westerg.	Meyerhof	Falkner	Shentu	Westerg.	Meyerhof	Falkner	Shentu
0.015	57.9	104.9	133.8	336.5	47.3	122.4	181.2	231
0.035	62.44	107.9	163.7	376.5	51.1	126.2	225.1	255.3
0.05	64.6	109.3	179.1	406.6	52.9	128	247.9	273.5
0.1	69.2	112.6	215.1	506.6	56.6	132.2	300.5	334.1
0.15	72.21	114.7	240.7	606.7	59.1	134.9	338.4	394.8
0.2	74.5	116.4	261.3	706.7	61	137.1	369	455.4
(1)*0.25	76.4	117.8	278.9	806.8	62.6	138.8	395.4	516.1
0.3	78	119	294.5	906.8	63.9	140.4	418.3	576.7
0.35	79.4	120	308.5	1006.9	65.1	141.7	439.3	637.3
0.4	80.7	121	321.4	1107	66.1	142.9	458.1	698
(1)* Find sample of calculations in Appendix (E)								

8.2.4 Edge and Corner Load Capacity

Table 8-3 shows the results of calculation of edge and corner load capacity of the two slabs by using Westergaard, Meyerhof. The calculation was performed using the properties indicated in table 8-1. Apart from the measured K-value, other K-values were assumed and the relevant load capacities were calculated.

Table 8-3: Calculated Edge and Corner Load for Various K-values

K (N/mm^3)	Plain Concrete Slab (150 mm Thickness)				SFRC Slab (125 mm Thickness)			
	Edge		Corner		Edge		Corner	
	Westerg.	Meyerhof	Westerg.	Meyerhof	Westerg.	Meyerhof	Westerg.	Meyerhof
0.015	33.7	64.9	34.6	39.2	27.6	76.1	28.2	46.2
0.035	36.5	67.5	36.6	41.2	29.9	79.4	29.1	48.7
0.05	37.9	68.8	36.2	42.1	31	81.1	29.5	49.9
0.1	40.7	71.6	37.4	44.3	33.4	84.7	30.5	52.7
0.15	42.6	73.5	38.3	45.7	34.9	87.1	31.3	54.5
0.2	44.1	74.9	39	46.9	36	88.9	31.9	56
0.25	45.3	76.2	39.7	47.8	37.1	90.5	32.4	57.1
(2)* 0.3	46.3	77.2	40.2	48.6	38	91.8	32.8	58.2
0.35	47.2	78.1	40.7	49.3	38.7	93	33.3	59.1
0.4	48	78.9	41.1	49.9	39.4	94.1	33.6	59.9
(2)* Find sample of calculations in Appendix (E)								

8.2.5 Deflection

Table 8-4 shows the results of calculating vertical interior, edge and corner deflection using relevant Westergaard load for the two slabs. The calculation was performed using Westergaard formulas. Apart from the measured K-value, other K-values were assumed and the relevant deflections were calculated.

Table 8-4 Deflections for Various K-values (Calculated by using Westergaard)

K- Values (N/mm^3)	Plain Concrete Slab (150 mm)			SFRC Slab (125 mm)		
	Interior (mm)	Edge (mm)	Corner (mm)	Interior (mm)	Edge (mm)	Corner (m m)
0.015	0.71	1.43	2.97	0.71	1.44	2.89
0.035	0.50	1.02	1.88	0.5	1.02	1.18
0.05	0.43	0.88	1.55	0.44	0.89	1.48
0.10	0.33	0.67	1.05	0.33	0.68	0.99
0.15	0.28	0.57	0.83	0.28	0.58	0.78
0.20	0.25	0.51	0.7	0.25	0.52	0.65
0.25	0.23	0.47	0.61	0.23	0.47	0.57
(3)* 0.30	0.21	0.44	0.55	0.22	0.44	0.5
0.35	0.20	0.42	0.50	0.2	0.42	0.46
0.40	0.19	0.40	0.46	0.19	0.40	0.42
(3)* Find sample of calculations in Appendix (E)						

8.3Comparison Between Theory and Practice

8.3.1 Westergaard

8.3.1.1 Load Capacity

Load capacities calculated using Westergaard model are compared to the *loads at first crack* because Westergaard formulas assume an *elastic* behaviour. The after crack toughness of SFRC cannot be considered; therefore the influence of the steel fiber can be considered only when it contributes to increase the first crack strength if any.

Figure 8-2 shows a comparison between actual measured first crack load and the calculated load using Westergaard for the SFRC slab. It was found that the actual measured values are greater than calculated values by about 510%, 375% and 490% for interior, edge and corner loads respectively.

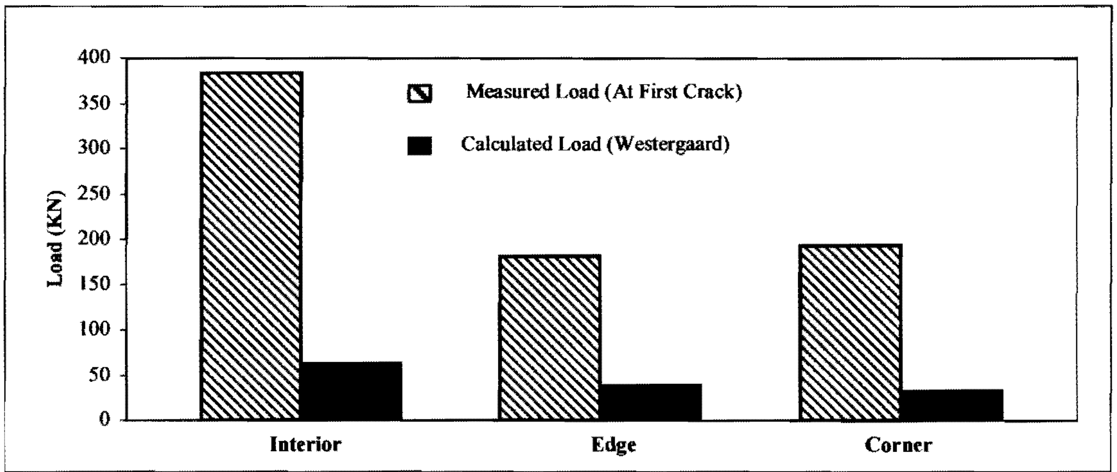

Figure (8-2): Measured Load and Westergaard Load for SFRC Slab

Figure 8-3 shows a comparison between actual measured first crack load and the calculated load using Westergaard for the plain concrete slab. It was found that the actual measured values are greater than calculated values by about 420%, 300% and 400% for interior, edge and corner loads respectively.

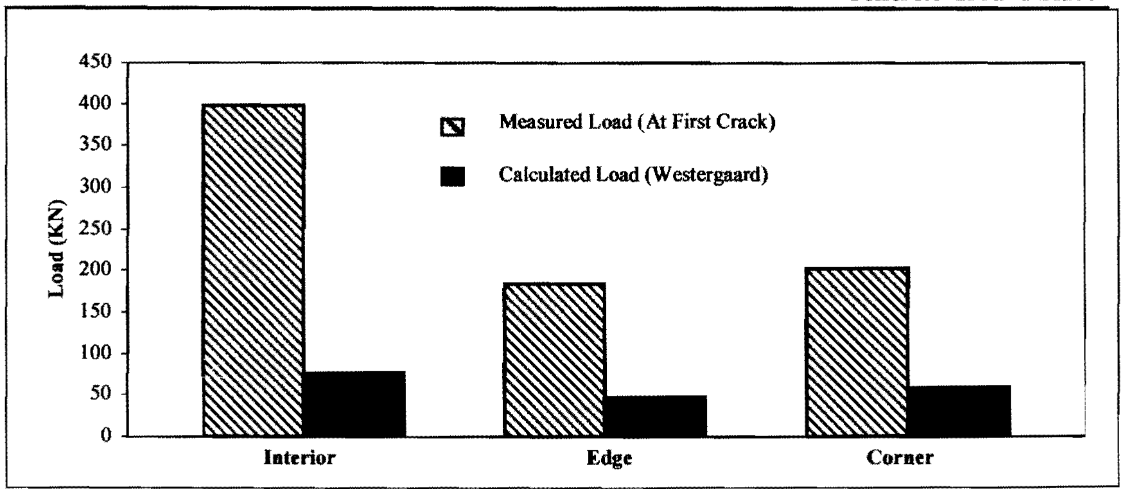

Figure 8-3: Measured Load and Westergaard Load for Plain Concrete Slab

The measured load capacities are very far from the calculated loads and this result correlates with results obtained by other researchers [77].

The hard foamed concrete subbase might influence the result. Figure 8-4 shows the relation between K-value and calculated load capacity. It is evident that there is a certain limit of K-value beyond which the increase in its value does not increase the calculated load capacity significantly. The reality might be different, because with increasing the sub grade reaction the slab and underlying layers tend to act as one unit thus increasing the load capacity dramatically.

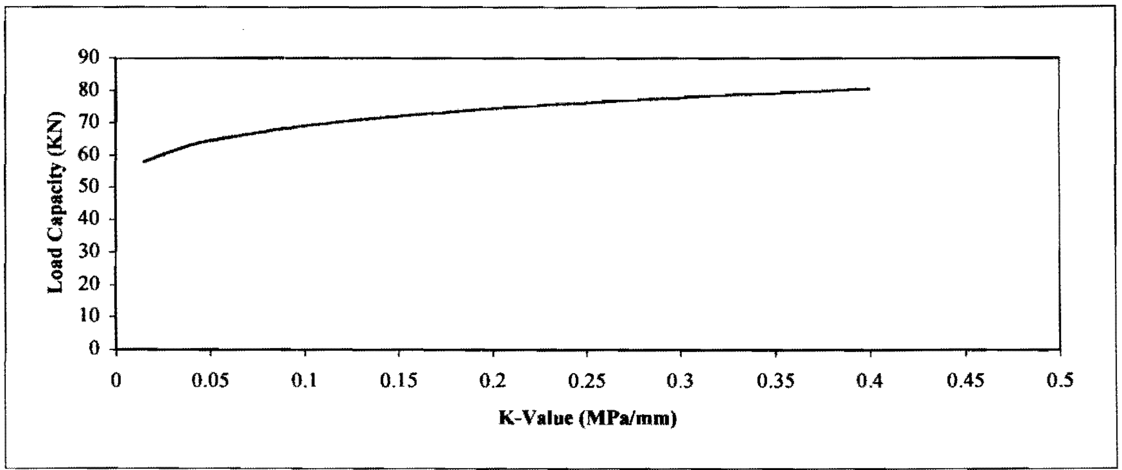

Figure 8-4: Sub grade Reaction-Load Capacity Relationship (Westergaard)

The theory of Westergaard was based on infinite slab. The dimensions of slabs used for this investigation might not be suitable for the comparison between Westergaard load and actual measured loads. There may be a certain minimum dimension required for the slabs for which better correlation could be obtained.

The first crack determination might also cause some disagreement. The first crack as defined (point at which load-deflection curve deviate from linearity) was not clearly found because the point of first crack as defined does not exist in the most of the cases. Therefore the estimated value might not be the real first crack.

The theoretical load capacity for the plain concrete slab is at most 18% more than the theoretical capacity for the SFRC slab while the measured loads were found to be approximately equal. This shows that the actual structural behaviour of the SFRC is different to that of plain concrete even prior to cracking. Westergaard formula doesn't seem to consider that behaviour and the SFRC is considered as plain concrete having higher first crack strength and less depth.

8.3.1.2 Deflection

The deflection calculated using Westergaard model is compared to the *deflection at first crack* as Westergaard formulas assume an *elastic* behaviour. The measured deflections were corrected by subtracting the seating deflections at the start of the load-deflection curve.

Figure 8-5 shows a comparison between actual measured first crack deflection at Westergaard load and the calculated deflection using Westergaard load for the SFRC slab. It was found that the actual measured values are greater than calculated values by about 150%, 60% and 115% for interior edge and corner loads respectively.

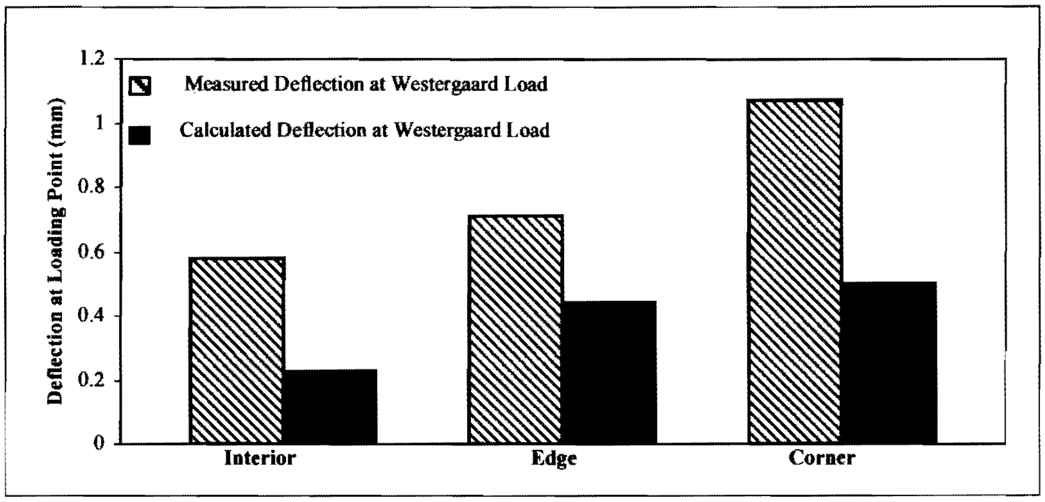

Figure 8-5: Calculated and Measured Deflections at Westergaard Load for the SFRC Slab.

Figure 8-6 shows a comparison between actual measured first crack deflection at Westergaard load and the calculated deflection using Westergaard load for the plain concrete slab. It was found that the actual measured values are greater than calculated values by about 130%, 130% and 75% for interior, edge and corner loads respectively.

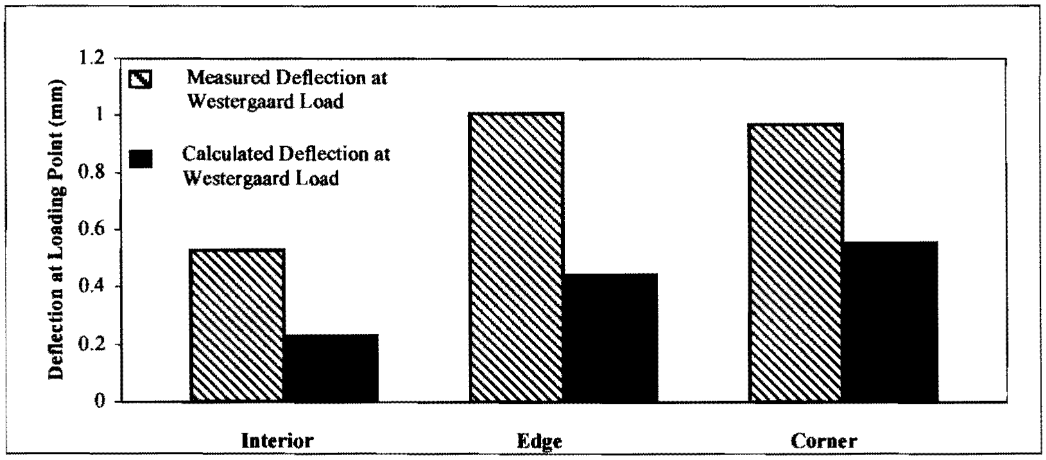

Figure 8-6: Calculated and Measured Deflections at Westergaard Load for the Plain Concrete Slab

For both slabs, the deflection values calculated using Westergaard is far less than that measured. This result was partially confirmed from the finite element analysis developed by Ioannides et al [83] which shows that slab requires certain dimension to yield deflections approximately equal to that of Westergaard. He found that the ratio of the least dimension to the radius of relative stiffness (L/l) should be equal or greater than a value of 8.0 for interior and edge loads and 5.0 for corner load. For smaller ratios, larger deflection values will take place with slab load testing. L/l are calculated as 8.2 (3000/367.9) and 7.4 (3000/407.4) for the SFRC and plain concrete slab respectively. These values indicate that limitations of Ioannides might needs further adjustment. In addition to that, the SFRC has different behaviour than that of plain concrete.

Table 8-4 shows that the elastic deflection calculated using Westergaard formula are approximately equal for both slabs, even they have different depths. Due to it's reduced thickness; the SFRC slab was expected to yield higher calculated deflection values. Once again, one could realize that Westergaard formulas are not suitable for the SFRC in the way that it underestimates the loads and corresponding deflection.

The K-value influences the calculated deflection for both the SFRC and plain concrete slabs. Both slabs were found to have approximately the same sensitivity to the K-value and that can be seen from table 8-4. Also it can be seen that increasing K-values in lower ranges results in a higher reduction to deflection than that of the higher range of K-values. Corner conditions were found to be the most sensitive to the above-mentioned phenomenon. For instance, an increase of K-value from 0.015 to 0.4 MPa/mm decreasing the corners deflection by about 7 times while the same increase in K-values results in a reduction of 3.5 times as to edge and interior deflection of both slabs.

8.3.1.3 Failure Characteristics

For interior loading, Westergaard estimated the radius of circumferential crack to be 1.9 of the slab radius of relative stiffness, which is about 700 and 774mm for the SFRC and plain concrete slabs respectively. The deflection profiles discussed in the previous chapter shows that the radius of crack is about 400 and 350mm for the SFRC and plain concrete slabs respectively, which is far less than that given by Westergaard

It was also found that the crack location for the corners loaded at 150 mm from its angle bisector is about 280 and 260 mm from the corner bisector for SFRC and plain concrete respectively. This again less by 16% and 26% from the calculated values for SFRC and plain concrete respectively. The following formula was given by Westergaard to give the radius of the crack from corner angle bisector [85]:

$$r = 2.38\sqrt{al} \quad \Longrightarrow \quad \text{Eq.8-1}$$

Where :
r = Crack radius.
a = Radius of loading plate.
l = Radius of relative stiffness.

8.3.2 Meyerhof

8.3.2.1 Load Capacity

Load capacities calculated using Meyerhof model is compared to the *Maximum loads "at failure"* because Meyerhof formulas assume an *elastic-plastic* behaviour for concrete. The after crack toughness of SFRC can now be taken into account.

Figure 8-7 shows a comparison between actual measured ultimate load and the calculated load using Meyerhof for the SFRC slab. It was found that the actual measured values are greater than calculated values by about 370%, 485% and 560% for interior, edge and corner loads respectively.

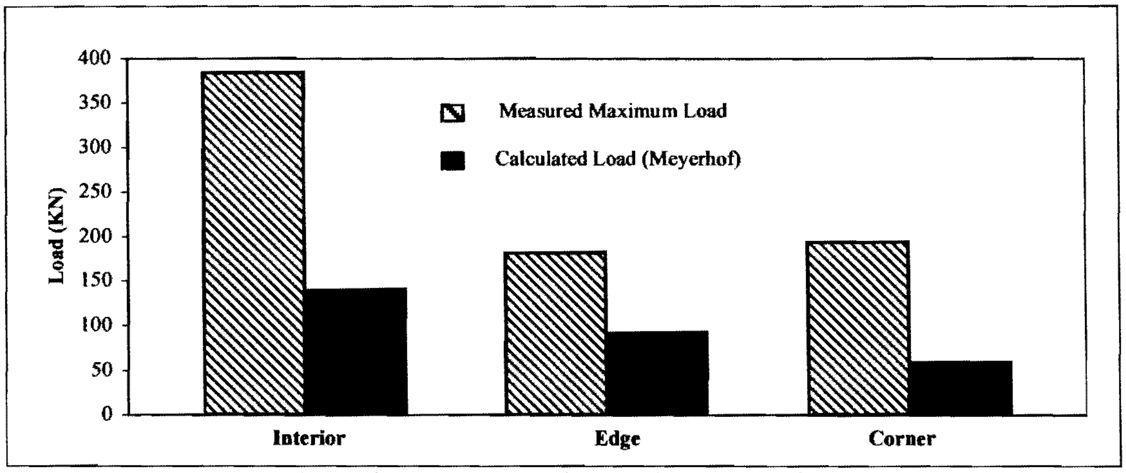

Figure 8-7: Measured Load and Meyerhof Load for SFRC Slab

Figure 8-8 shows a comparison between the actual measured ultimate load and the calculated load using Meyerhof for the plain concrete slab. It was found that the actual measured values are greater than calculated values by about 520%, 560% and 800% for interior edge and corner respectively.

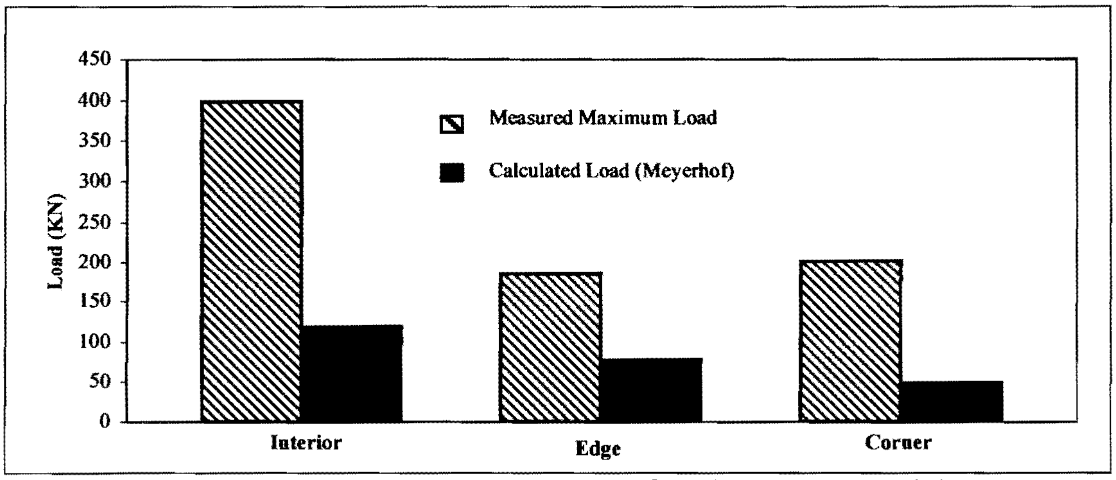

Figure 8-8: Measured Load and Meyerhof Load for Plain Concrete Slab

Obviously, the measured load capacities are far from agreement with the calculated loads. Although the difference between the calculated load and measured loads is significant, the difference is smaller for the SFRC slab.

For corner load, the British Concrete Society Technical Report No. 34 [39] does not consider the after crack toughness of the SFRC and the following equation was used to calculate the moment of resistance for both plain concrete and SFRC slabs.

$$M_o = \frac{f_{ct} b h^2}{6} \Longrightarrow \text{Eq.8-2}$$

Where :

M_o = Moment of resistance.

f_{ct} = Flexural strength.

b, h = Width and depth respectively.

Steel fiber manufacturer design catalogues [11] suggests the inclusion of the SFRC toughness by changing the f_{ct} to be f_{ct} $[1+R_{e,3}]$. Doing this for the SFRC slab improves the correlation between calculated and measured values, but the difference is still far from acceptable.

The calculated load capacity increases with the increase of the K-value, which implies similar limitations to that, discussed for Westergaard in section 8.3.1.1.

8.3.2.2 Deflection

Although Meyerhof did not give deflection formulas, the model was developed and adjusted using data from existing airfield pavements. The calculated deflection corresponding to the Meyerhof load should agree with actual deflection corresponding to that load. Deflection in the case of Meyerhof model is elastic-plastic deflection.

Actual deflections *at Meyerhof load* are read-off from the load-deflection curves. For the SFRC slab the deflections corresponding Meyerhof's interior, edge and corner are 0.61, 2.1 and 2.1 mm respectively while deflections for plain concrete slab are 0.68, 1.4 and 1.4mm for interior edge and corner respectively.

8.3.2.3 Failure Characteristics

Figure 8-9 shows the failure mechanism considered by Meyerhof model [86]. This mode of failure "fan failure" is assumed for a large slab resting on linear elastic (Winkler) subgrade (imply complete contact between slab and sub grade). For the slabs under consideration, the mode of failure in figure 8-9 was not observed. This might be the reason for the disagreement between the calculated loads using Meyerhof model and actual measured loads. Once again, the hard subgrade might have influenced that.

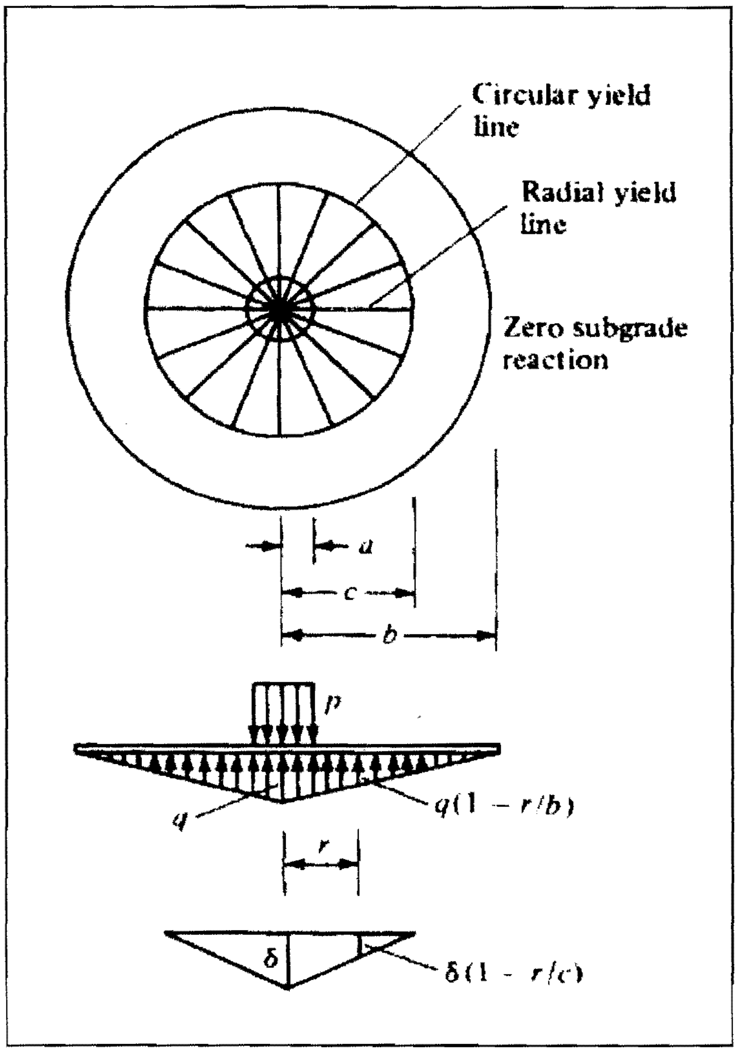

Figure 8-9 Failure Mechanism for Meyerhof Model

Although the "fan failure" was not observed, the deflection profiles obtained for both slabs indicate that a tiny crack might have taken place and that the radius of that crack is 400 and 350 mm for the SFRC and plain concrete slabs respectively. The following equation was given by Meyerhof to calculate the theoretical radius of circumferential crack [85]:

$$r = 1.63\sqrt{al} \Longrightarrow \text{Eq.8-3}$$

Where :

$r =$ Crack radius.

$a =$ Radius of loading plate.

$l =$ Radius of relative stiffness.

The calculated radius is equal to 230 for the SFRC slab and 240 for plain concrete slab. This is still less than the actual values but it is a better estimation than that given by Westergaard.

8.3.3 Falkner et al

8.3.3.1 Load Capacity

Load capacities calculated using the model of Falkner et al are compared to the *Maximum loads "at failure"* because the formulas assumed by Falkner et al are based on two limit states. One is the limit state of cracking and the other is the ultimate failure state. For the calculation of the first cracking load, Westergaard equation was used. For the plastic behaviour (after cracking) the plastic design principles were used.

Figure 8-10 shows a comparison between actual measured ultimate load and the calculated load using Falkner et al for the SFRC and plain concrete slabs. It was found that the actual measured values are greater than calculated values by about 66% and 160% for the SFRC and plain concrete slabs respectively.

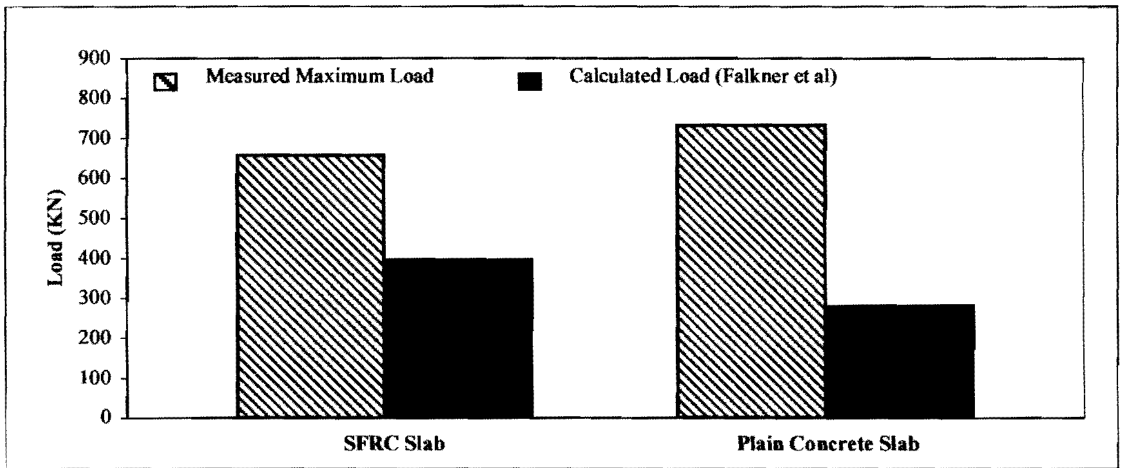

Figure 8-10: Measured Load and Falkner et al Load for SFRC and Plain Concrete Slabs

Although the measured load capacities do not agree with the calculated loads, less difference is found than that obtained from Westergaard and Meyerhof. Falkner model was adjusted using data from (3 x 3m) slabs so the effect of the slab size on calculated loads is eliminated.

Better agreement between the calculated load and measured load for the SFRC slab than the plain concrete slab, imply that Falkner et al model is better in considering the after cracking behaviour for the SFRC.

The calculated load capacity (using Falkner et al model) increases with the increase of the K-value. As discussed before, the stiffness of the support could have influenced the correlation between measured and calculated values.

Apart from considering the three load case, analytical models used for designing SFRC ground slabs should have the capability to include the after cracking toughness imparted by addition of steel fibers. Although Falkner et al model considers for the after cracking toughness, it does not take edge and corner load into account beside it does not calculate the ultimate deflection.

8.3.3.2 Failure Characteristics

Figure 8-11 shows the failure criterion considered by the model of Falkner et al [73]. The graph shows three stages of failure and the slab sketches on the right hand shows the physical consequences of each stage. This type of failure was observed for the tested slabs and the load-deflection diagrams for the slabs seem to follow a similar sequence to that considered by Falkner et al. The disagreement of the calculated load capacities using Falkner et al and the measured values might basically relate to the difference of mode of failure actually obtained and that considered by the used model.

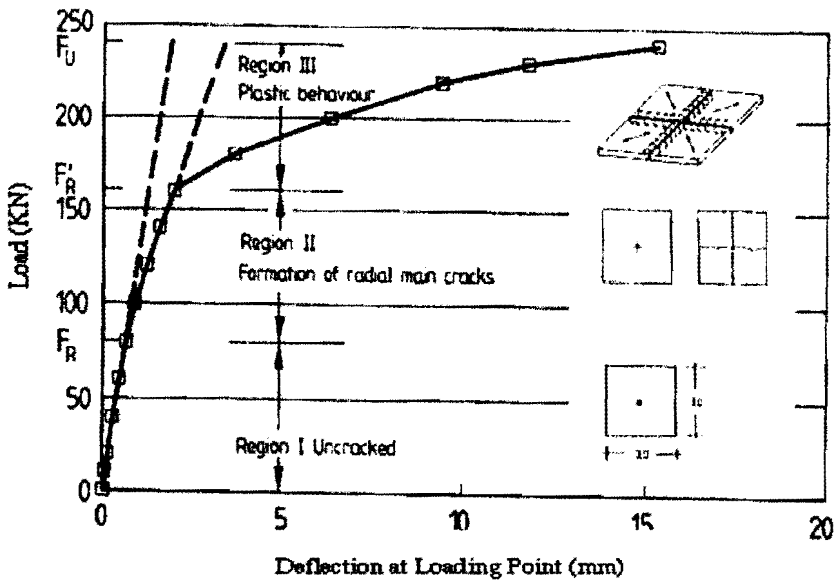

Figure 8-11: Principle Load-Deflection Behaviour and Failure Mechanism (Falkner et al)

8.3.4 Shentu et al

Load capacities calculated using the model of Shentu et al are compared to the *Maximum loads "at failure"* because Shentu et al formulas assume an *elastic-plastic* behaviour. In this comparison, the after crack toughness of SFRC can be taken into account.

Figure 8-12 shows a comparison between actual measured ultimate loads and the calculated load using Shentu et al for the SFRC and plain concrete slabs. It was found that the actual measured load for SFRC slab is greater than calculated load by about 27% while the calculated load for the plain concrete slab is 10% greater than the measured load.

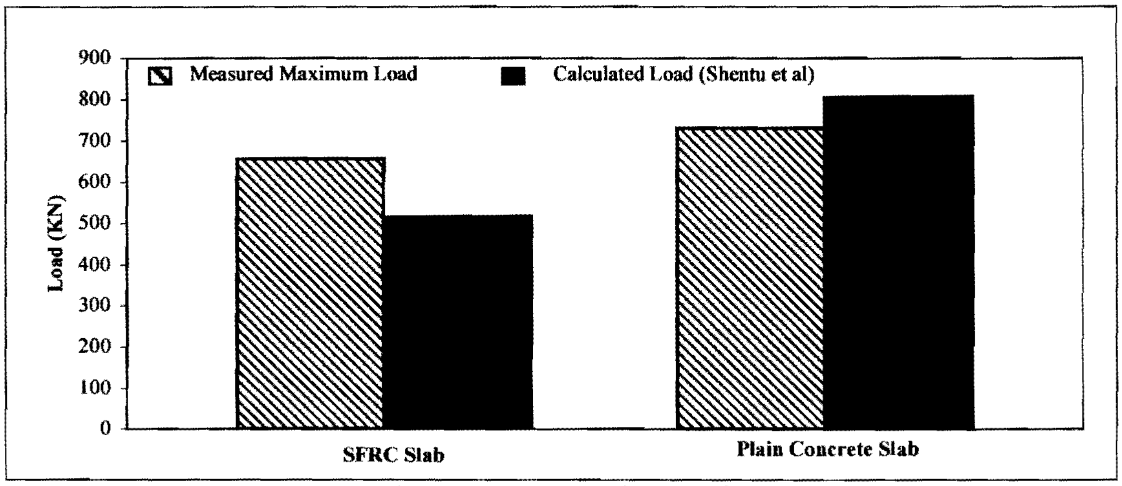

Figure 8-12: Measured Load and Shentu et al Load for SFRC and Plain Concrete Slabs

Reasonably agreed calculated and measured values are obtained. The ultimate load as given by Shentu et al is directly related to sub grade reaction, radius of load, direct tensile strength and the depth of slab and inversely related to modulus of elasticity. The low calculated value for the SFRC is due to the slightly higher modulus of elasticity and the smaller depth while the opposite is true for the plain concrete slab.

Calculated load capacity increases with the increase of the K-value. Figure 8-13 shows that the load-K-value relationship is linear unlike that found for Westergaard Meyerhof and Falkner et al. That might explain the higher loads calculated using Shentu et al compared to loads calculated using the other three models.

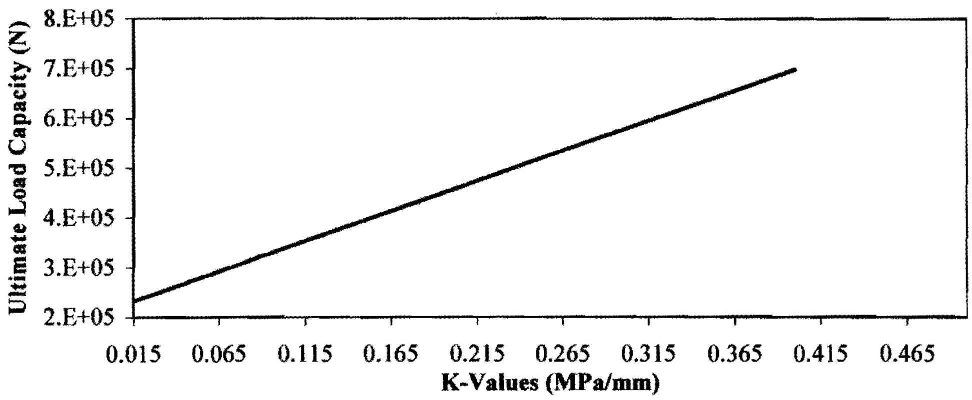

Figure 8-13: Sub grade Reaction-Load Capacity Relationship (Shentu et al)

The model of Shentu et al is found not to be sensitive to the slab width, unlike the other three models. With this model, slabs with different dimensions but having similar other factor will yield the same load capacity. This model uses the uniaxial tensile strength as one of its inputs and the assessment of uniaxial strength is found difficult. The model cannot be used to assess the edge and corner load or any deflections.

8.4 Conclusions

- The calculated load capacities for the two slabs were found to differ significantly from values calculated using the models of Westergaard and Meyerhof. Some correlation was found with the values calculated using Falkner et al and Shentu et al models.
- Westergaard model is found not suitable for the SFRC slabs, because it does not consider the after cracking behaviour.
- Meyerhof model seems to be the most suitable to apply for the SFRC slabs because the after cracking behaviour can be taken into account and the three load cases (interior, edge and corner) can be considered. The shortcoming of this model is that the deflection is not modeled.
- The model of Falkner et al takes the improved toughness of SFRC into account and it gives better estimation than using Meyerhof. Edge and corner loads and deflection are not modeled. Further research is needed to improve the model to consider the three load cases and deflection.
- Based on the measured and calculated results, the model of Shentu et al is the most suitable for predicting the load capacity of ground slabs.