

INFERENCES OF α -STABLE DISTRIBUTION OF THE UNDERLYING NOISE COMPONENTS IN GEODETIC DATA

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ABSTRACT

We investigate the use of distribution functions to characterise the geophysical signals and noise components embedded in the geodetic Very Long Baseline Interferometry (VLBI) data sets across some of the International VLBI Service (IVS) stations. The rationale of using α -stable distributions as a tool to model the noise components in geodetic observables is due to the existence of impulsive signals/noise bursts (which often take the form of excursions with intermittent occurrences) in the data sets suggesting deviations from Gaussian distribution. A deviation from Gaussian distribution type would therefore suggest that statistical techniques such as least squares analysis, often used for analyzing the geodetic data (which are often based on Gaussian assumptions) could not be robust. In this paper, the properties of a long-range α -stable distribution with long tails and infinite moments in geodetic data are investigated by way of statistically testing their distribution using a family of stable distributions. The choice of stable distributions is based on the ease with which the statistical properties of the non-Gaussian processes are defined. Results indicate that the independent geophysical noise components reconstructed from geodetic VLBI baseline data exhibit distributions that have asymptotic power-law decay (albeit variable power indices) whose underlying process can be modelled as a long-range dependent process with an α -stable distribution (i.e., the stable varieties have small characteristic exponents).

Introduction

Recent analysis of geodetic data demonstrated that the oscillatory modes of geophysical signals exhibit self-similar behaviour (Botai et al., 2008). This scaling behaviour could be viewed as a manifestation of scale-invariant non-linear dynamics whose fractal structures and multi-fractal statistics are typical of a turbulent atmosphere and the Earth's variable interior and complex topography.

Fundamental geodetic observables (i.e., the Earth's shape, the Earth's gravity field and the Earth's rotation) are provided by modern geodesy under the framework of the Global Geodetic Observing System (GGOS); reference to Plag and Pearlman, (2009) for further details on GGOS. These observables contribute to improve understanding of geodynamics, geo-hazards, the global hydrological cycle, global change, the dynamics of the atmosphere and oceans as well as supporting many societal applications that depend on accurate positions and reference frames.

In particular, space geodetic observables derived from space geodetic techniques such as Global Navigation Satellite Systems (GNSS), Satellite Laser Ranging (SLR), Very Long Baseline Interferometry

(VLBI), Lunar Laser Ranging (LLR) and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) provide a global picture of the Earth. This is achieved through observing:

- the changes of the surface geometry of the Earth due to the horizontal and vertical deformation of land surfaces and variations of the ocean surface and ice cover;
- fluctuations of the Earth (rotation, precession, polar motion and nutation); and
- variations of the gravity field and centre of mass.

A summary of geodetic parameters related to the system Earth reported by Rothacher, (2002) is contained within Table 1.

Furthermore, global monitoring of the Earth from space plays a significant role in understanding the processes in the complex Earth system. A robust knowledge of processes could aid in the development of geophysical models describing past and present events based on a priori data which would then be used to forecast for example future natural hazards. In Rothacher et al., (2010), the description and modelling of geophysical processes in the Earth system is reported to entail the development of data collection

Table 1. Geodetic parameter groups related to system Earth as reported in Rothacher (2002).

	Parameter type	VLBI	GPS	SLR	DORIS	LLR
CRS	Quasar positions	X				
	Orbits (satellites, moon)		X	X	X	X
	Nutation	X				X
	Nutation rates	X	X	X	X	X
EOP	UT1-UTC	X				
	LOD	X	X	X	X	X
	Polar motion	X	X	X	X	X
TRF	Station positions	X	X	X	X	X
	Geocentre		X	X	X	
Gravity field	Low degree		X	X	X	X
Atmosphere	Troposphere	X	X		X	
	Ionosphere	X	X		X	

and an earth management system as well as robust and tractable data analysis strategies. These analysis methods may provide consistent and near-real time integrated geodetic time series which are referenced to an extremely accurate and stable reference frame.

Geodetic measurements are often characterised by complex and delicate operations that are prone to numerous problems such as instrumental limitations, atmospheric biases and poor network geometry. This leads to systematic deficiencies in sampling, both in time and space and measurement noises. As a result, the observables or estimated parameters will rarely show any smoothness at any scale and therefore any comparison with theoretical model output would be difficult. In addition, the inherent noises and irregular observation space may produce an error spectrum in the geodetic solutions that have a high degree and order of dependencies and cross-correlations.

Generally, many problems in the field of geodetic-geophysical research reduce to decomposing the geodetic record and separating weak geophysical components embedded in the geodetic data. As a result, isolating the noise components in geodetic data is critical for the interpretation and modelling of geophysically interesting signals embedded in the data (Beavan, 2005). For example, in the studies reported by Riva et al., (2009), it was demonstrated that separation of geophysical signals (i.e., the Glacial Isostatic Adjustment Signal (GIAS) associated with the meltdown of the Late-Pleistocene ice sheets and other geophysical signals such as gravity and crustal deformation) embedded in geodetic data was problematic.

In order to extract information from geodetic data, application of appropriate models and robust mathematical statistical methods are required. In particular, the procedure used to compute geodetic parameters such as tropospheric delay, station coordinates, baseline lengths and Earth Orientation Parameters (EOPs) ought to be formulated in a tractable manner. Furthermore, the algorithms used for processing the parameters should be quantitatively characterised by use of robust probabilistic models (including characteristic and distribution functions).

To date, research problems in space geodesy are mostly dominated by Gaussian assumptions during parameter estimation and this could be attributed to the,

- validity of the Gaussian model for a large number of data sources based on the Central Limit Theorem;
- well known and mature procedures of designing algorithms based on Gaussian assumption; and
- simple linear functional form of the resulting algorithm.

While the use of simple models in parameter estimation from geodetic data has been satisfactory in many applications to date, future accuracy requirements (Niell, 2005) make it necessary to quantitatively examine the contribution of complex models (which take into account non-linear and nonstationary structure in geodetic data) during parameter estimation.

Concerns regarding the suitability of linear/Gaussian models emerged due to certain factors, including the increase in the spatial-temporal span of geodetic data sets. In particular, several geodetic data sets have provided evidence for error sources that introduce large temporal correlations into the data (Williams, 2003). For example, the analysis of the characteristic temporal correlations based on noise models in time series of 236 GPS receiver position changes for data spanning between 3.5 and ten years operating in Southern California and Southern Nevada was reported by Beavan, (2005). Results reported in Beavan, (2005) demonstrated that the geodetic record exhibits either and/or flicker, random walk, power law, Gauss-Markov and seasonal noise. In addition, Amiri-Simkooei et al., (2007) used Least Squares Variance Component Estimation (LS-VCE) and determined that the noise components in the time series of all components of position estimates could best be characterised by a combination of white and flicker noise.

Mao et al., (1999) assessed the noise characteristics in time series of daily position estimates for 23 globally distributed GPS stations with three years of data, using spectral analysis and Maximum Likelihood Estimation (MLE). From their analysis, a combination of white and flicker noise appear to be the best model for the noise characteristics of all three position components.

Both white and flicker noise amplitudes were found to be smallest in the north component and largest in the vertical component. The white noise part of the vertical component was higher for tropical stations ($\pm 23^\circ$ latitude) compared to mid-latitude stations. These observations were confirmed by Williams et al., (2004) who used MLE analyse for the noise content present in a total of 954 GPS position time series from 414 individual sites in nine different GPS solutions.

The presence of a mixture of noise (white and/or coloured noise) in geodetic data sets suggests that a power-law process may be used to describe a statistical model for the geophysical signals (and noise), see Williams, (2003). Such models appear in a large number of applications in finance (Mantegna and Stanley, 2000), traffic network (Abery and Veitch, 1998), geophysics (Davies et al., 1999) and atmospheric science (Varotsos et al., 2009). Furthermore, several geodetic data sets have evidenced power-law scaling behaviour (Botai et al., 2008; Botai et al., 2010). As a result, the decaying dependence (long memory) observed in geodetic parameters may not be adequately characterised in terms of the traditional models such as the Gaussian model and auto-regressive moving average models because of their inability to capture the characteristics of space-time evolution.

While models for long memory (and therefore self-similarity) processes (which have infinite variance) have been reported in the literature, applying these models to real data is still lacking. In order to explore the characteristics of the underlying geophysical signals and noise components in the geodetic data, assessing the stability of the inherent frequency distribution is often required. In the present research, the properties of a long-range α -stable distribution with long tails and infinite moments that characterize the noise components in geodetic data are investigated by way of statistically testing their distribution. A notable example of application of Lévy distributions has been in the field of geomagnetism where the frequency of geomagnetic reversals seemed to follow a Lévy distribution. Furthermore, Scafetta and Bruce, (2008) linked the statistical structure (the Lévy distributions) evident in the intermittency of solar flare data to the intrinsic fluctuations in the Earth's temperature.

In particular, sequences of long-range dependent geodetic data time series whose distributions exhibit a power decay i.e., $\{q, \alpha\}$ -stable distributions are studied; here q controls the dependence. Long-range dependent $\{q, \alpha\}$ -stable distributions often arise in non-linear and nonstationary processes. The purpose of the present paper is to present a generalised methodology of describing the probability distributions of independent noise components embedded in geodetic data. Emphasis is on establishing properties of α -stable distributions which describe the noise components embedded in geodetic data. The outline of this paper is as follows; some aspects of α -stable distribution and statistical methods of testing for α -stable distribution are

presented in Section 2. Analysis method and data set utilised are presented in Section 3. In Section 4, results from the α -stable distribution model fit into the independent components in each of the seven VLBI derived global baseline data are presented and discussed. Concluding remarks are given in Section 5.

Statistical testing for α -stable distributions

The α -stable distribution or generally stable distribution was developed by Paul Lévy (see Borak et al., 2005 and others therein). The α -stable distributions are often suited for modelling impulsive phenomena. Many physical phenomena are non-Gaussian and in cases where the observed data have frequently occurring extreme values; they could be modelled as a realisation of a random process with an α -stable distribution. Simple 1-D α -stable distributions are typically heavy-tailed distributions with characteristic functions (this is because their densities and distribution functions are not known in closed form) given by Equation (1).

$$\Phi(\rho) = E[e^{j\rho x}] \quad (1)$$

$$= \begin{cases} e^{\{j\theta v - |\mu\theta|^\alpha (1 - j\beta \frac{\theta}{|\theta|} \tan(\frac{\pi\alpha}{2}))\}} & \text{if } \alpha \neq 1. \\ e^{\{j\theta v - |\mu\theta|^\alpha (1 - j\beta \frac{2}{\pi} \frac{\theta}{|\theta|} \ln|\theta|)\}} & \text{if } \alpha = 1. \end{cases}$$

In Equation (1), the stable distribution is specified by the parameter set $S\alpha\{\alpha, \beta, v, \gamma\}$. Here, the stable characteristic exponent ($0 < \alpha \leq 2$) characterises the tail of the distribution (the lower the value of α , the heavier the tail and the greater the probability of extreme values or 'spikes'), the skewness parameter ($-1 < \beta < 1$) is also called the symmetry parameter or index of distribution. If $\beta=0$, the distribution is symmetric (Symmetric α -Stable: $S\alpha S$), otherwise the distribution is positively (to the right) and/or negatively (to the left) skewed. The spread ($v > 0$) and location ($-\alpha < \gamma < \alpha$) parameters measure dispersion (determines the spread of the density) around its location parameter. For a normal distribution, γ is equivalent to the mean while the dispersion agrees with the variance of the Gaussian density: it equals to half the variance when $\alpha = 2$.

The family of α -stable distributions is extensive. Three notable subclasses of interest are:

- Gaussian distribution, $N\{\mu, \sigma^2\}$ which are often characterised by the parameter set $S\alpha\{2, 0, 2-0.5\sigma, \mu\}$;
- a Cauchy distribution with parameter set $S\alpha\{1, 0, v, \gamma\}$; and
- α -stable Lévy distribution by the set $S\alpha\{0.5, 0, v, \gamma\}$. Note also that α -stable distributions lack moments of order larger than α ; for $\alpha < 2$, there are no second-order statistics. For further details on the theory of

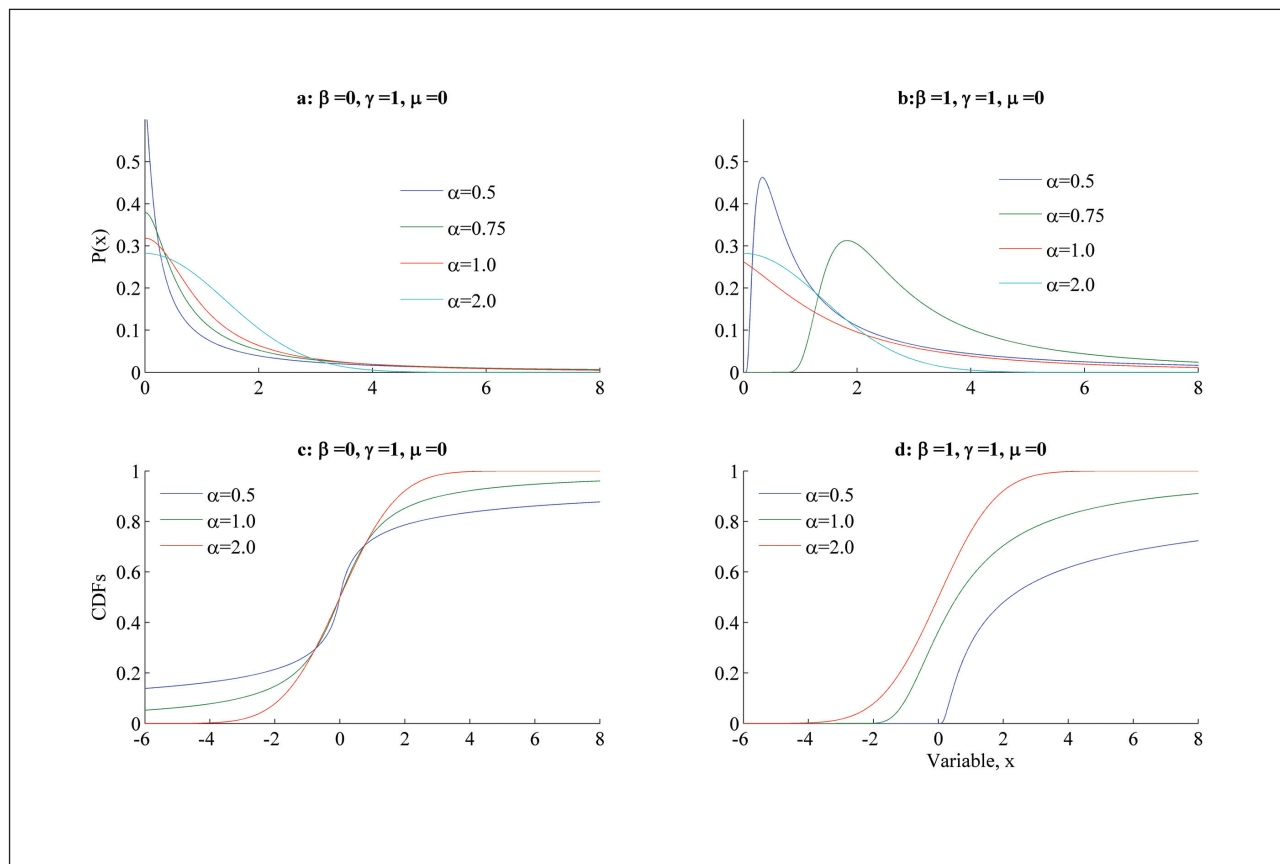


Figure 1. SaS (a) and asymmetric (b) α -stable probability density function (P) with varying characteristic exponent. SaS (c) and asymmetric (d) α -stable cumulative density function (CDF).

stable distributions, refer to Samorodnitsky and Taqqu, (1994) and Nikias and Shao, (1995) and references therein. In the present research, we focus on the α -stable distributions: especially, the α -stable Lévy distribution. This particular distribution is a continuous-time stochastic process $\Gamma t | t \geq 0$ which exhibits unique properties such as having independent and stationary increments (or independent and identical distribution) and admits cadlag (right continuous with left finite limits) modification, see for example Cont and Tankov, (2004) and references therein. Figure 1 depicts probability density functions for typical simulations of symmetric

- and asymmetric
- α -stable distributions for different values of α .

From Figure 1, it can be observed that the simulated α -stable distribution exhibits irregularities that are quite different from a Gaussian process. While the densities of the SaS maintain similarities to the Gaussian probability density functions, specific differences among the family of α -stable distributions are evident in the rate of the decay of the tails. In particular, the Gaussian density decays exponentially at its tails while non-Gaussian stable distributions decay algebraically.

Data and analysis procedure

Data sets and data pre-processing

Geodetic baseline data increments from seven International VLBI Service (IVS) stations, with relatively high quality and a long history of data sets spanning between 1990 and 2010 were considered in the present study. As depicted in Figure 2 the IVS stations were selected to ensure that there are good geometrical and global distributions. The geodetic baseline data sets were filtered using a modified adaptive filtering method described in Wessel et al., (2000). The purpose of using the adaptive filtering is to exclude possible artefacts from the baseline (often caused by inherent systematic differences in the solutions). Furthermore, polynomial trend (degree 1) and the seasonal components (period 4) were removed based on the moving average technique reported in Weron, (2007).

Extracting high energy-noise components in geodetic data

In the present study, we propose a combination of Empirical Mode Decomposition (EMD) and Independent Component Analysis (ICA) to isolate the frequency components in geodetic baseline data (hereafter BL). Firstly, the data are decomposed into spectrally independent oscillatory modes called the Intrinsic Mode Functions (IMFs) based on the Ensemble EMD (EEMD)

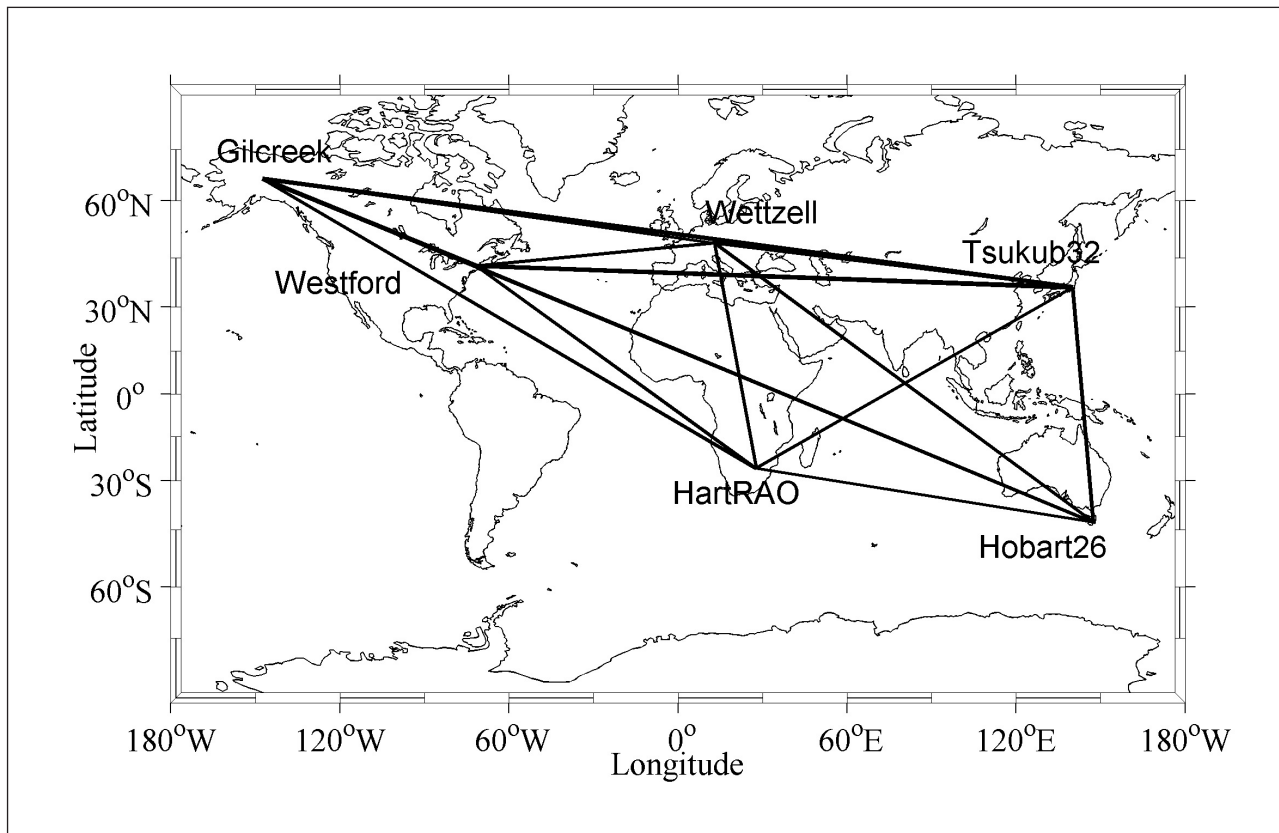


Figure 2. Global distribution of International VLBI Service (IVS) stations and some baselines.

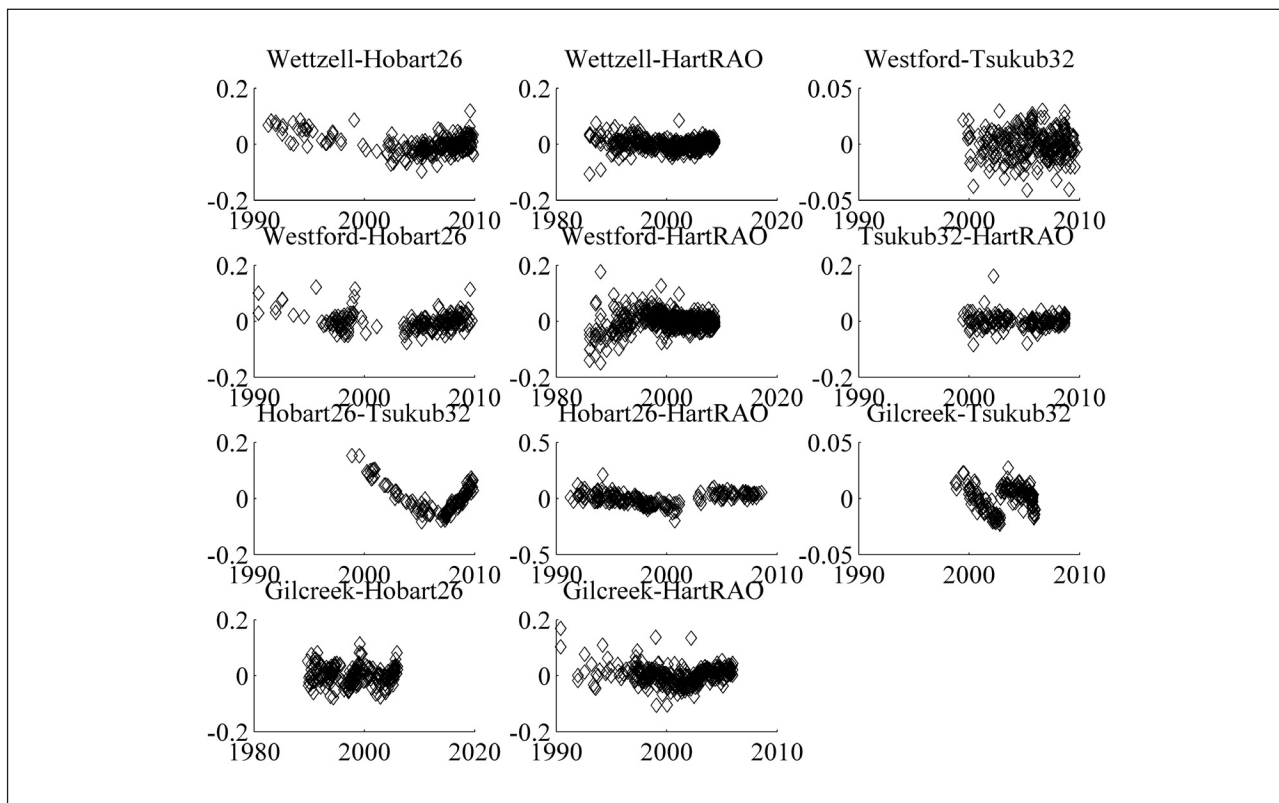


Figure 3. VLBI baseline lengths (determined with geodetic Calc/Solve analysis software) obtained from the Goddard Space Flight Center, Maryland, USA; an IVS analysis centre.

Table 2. Statistical properties of Very Long Baseline Interferometry derived baselines

	Number of Data points		WRMS [mm]		Slope [mm/yr]	
	C/S	OC	C/S	OC	C/S	OC
Gilcreek-HartRAO	343	304	18.23	21.08	3.19 ± 0.55	2.58 ± 0.74
Westford-HartRAO	514	460	17.51	18.22	11.23 ± 0.18	11.34 ± 0.22
Tsukub32-HartRAO	171	157	13.62	16.44	10.82 ± 0.43	-10.84 ± 0.53
Westford-Tsukub32	263	241	12.44	9.85	0.52 ± 0.30	-1.19 ± 0.26
Gilcreek-Hobart26	221	185	20.89	24.4	-38.74 ± 0.51	-38.55 ± 0.69
Tsukub32-Hobart26	125	111	10.9	13.01	-46.12 ± 0.47	-46.65 ± 0.54
HartRAO-Hobart26	264	195	21.63	27.59	33.04 ± 0.25	33.20 ± 0.38
HartRAO-Wetzell	469	419	11.26	12.16	-3.41 ± 0.12	-3.01 ± 0.15
Wetzell-Hobart26	240	212	19.19	21.36	-8.69 ± 0.40	-8.30 ± 0.55
Westford-Hobart26	190	167	12.64	9.85	-9.22 ± 0.64	-9.56 ± 0.42
Gilcreek-Tsukub32	164	146	6.6	6.34	4.82 ± 0.58	4.91 ± 0.53

Statistical calculations use Calc/Solve (C/S) or OCCAM (OC) software

algorithm which has been recently introduced by Zhaohua and Huang, (2009). This is a more robust noise-assisted version of the EMD algorithm developed by Huang et al., (1998) and is available online at <http://www.rcada.ncu.edu.tw/>. This MATLAB algorithm defines the IMFs for an ensemble of trials, each one obtained by applying EMD to the BL with an independent, identically distributed (iid) for the same standard deviation (σ_{iid}) where $\sigma_{iid} = \lambda \sigma_0$. Here, the ratio between the BL (σ_0) and iid is referred to as the noise parameter (hereafter, λ). The resulting IMFs are assumed to be the noise components of interest which are generally associated with high frequency IMFs. As a result, only those IMFs with high noise contamination ought to be used in the following ICA step. Two methods of selecting a physically significant IMF to be used in the ICA are employed. In the first method, the IMFs are selected based on the level of noise contamination in the sample. If the IMF₁ is assumed to be highly contaminated, then, its amplitude could be used as a reference of the amplitude of the noise contaminated IMFs. Statistically independent IMFs are extracted by selecting only those IMFs whose global energy is at most 50% of the energy of IMF₁. In the second criteria, ICA reduces the dimensionality of the IMFs adaptively.

Given that the number of statistically independent IMFs is now less than the original IMFs, ICA generally merges the statistically dependent components into the same group and therefore reduces the dimensional space of IMFs. Thus ICA is a statistical technique that decomposes a series of observations into a linear combination of non-Gaussian random variables that are highly independent (Hyvärinen, 1999). Nonlinear components imbedded in BLs were extracted adaptively as described in Botai et al., (2010) and references therein.

Results and discussion

Raw geodetic parameters exhibit power-scaling (their autocorrelation function decays rapidly) and therefore poses infinite variance. A plot of the 11 VLBI BL data sets (see Figure 3) depicts the presence of spikes suggesting that the underlying process is driven by non-Gaussian processes. The sampling density of the VLBI BL data varies between 150 and 500 data points spanning the period between 1998 and 2010.

The statistical properties of the BL as derived from Calc/Solve and OCCAM software contributed to the IVS by Goddard Space Flight Center (GSCF), VLBI analysis centre are tabulated in Table 2. The BL are determined using the 2000.0 as the reference epoch. From Table 2,

Table 3. α -Stable distribution parameters in highly fluctuating components in VLBI baselines

	IMF ₁		IMF ₁		IMF ₂		IMF ₂	
	α	β	γ	ν	α	β	γ	ν
Wetzell-Hobart26	2	0.01	0.01	0	1.92	0.42	0.01	0
Wetzell-HartRAO	-	-	-	-	1.62	-0.05	0	0
Westford-Tsukub32	2	0.6	0.01	0	1.74	0.43	0	0
Westford-Hobart26	2	-0.5	0.01	0	1.63	-0.03	0.01	0
Westford-HartRAO	-	-	-	-	1.53	-0.03	0.01	0
Tsukub32-HartRAO	1.69	-0.24	0.01	0	1.34	-0.17	0.01	0
Hobart26-Tsukub32	2	1	0.01	0	2	-0.23	0	0
Hobart32-HartRAO	1.98	-1	0.02	0	1.72	-0.03	0.01	0
Gilcreek-Tsukub32	2	-1	0	0	2	0.63	0	0
Gilcreek-Hobart26	2	-1	0.02	0	2	-0.76	0.01	0
Gilcreek-HartRAO	-	-	-	-	1.4	-0.08	0.01	0

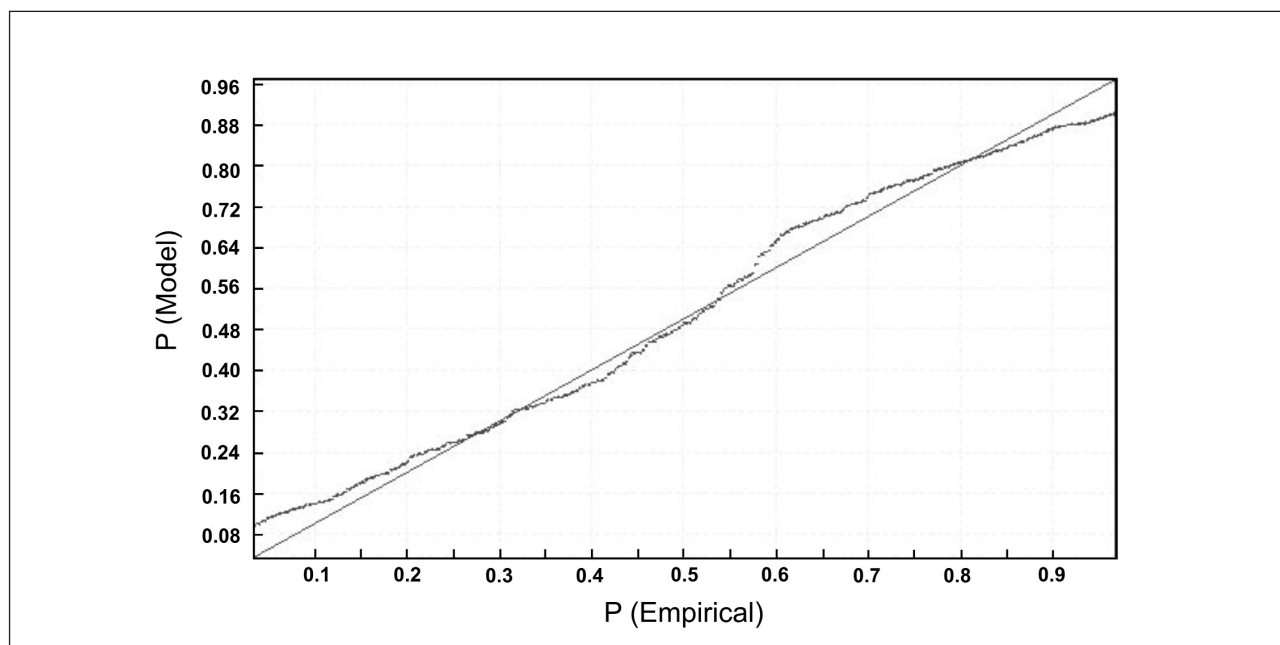


Figure 4. The p-p plot of the empirical (Cauchy) α -stable distribution and the independent noise components (e.g., IMF1) in the Wettzell-Hobart baseline data. Noise components derived from other baselines exhibit similar shape but with α -stable distribution members varying between Lévy and Cauchy distributions.

there are noticeable differences in the Weighted Root Mean Square (WRMS) values, which could be attributed to systematic differences in the sampling interval and individual observations as well as differences in the theoretical models used in computing the geodetic observation. Furthermore, the WRMS vary across the analysis software in all the stations. These differences could be attributed to the differences in data quality checks between the two IVS analysis centres as well the different geophysical model parameterisations during VLBI data analysis. In general, the statistics show close correlation and this suggests that the methodology of computing the BL results is robust (i.e., minima systematic differences in solutions from the different analysis software and the IVS analysis centres).

In order to infer whether the distribution of the noise component(s) in geodetic data is consistent with α -stable distribution, a p-p (probability-probability) plot is fitted to the data. In this analysis, the p-p plot is used to effectively describe the tail distribution of data relative to the simulated α -stable distribution. The p-p plot (based on the Cauchy distribution) in Figure 4 shows that the BL data closely fit the theoretical line, suggesting that the BL data fit the α -stable distributions including the tails.

To validate the results of the p-p plot, the sample characteristic function method is used to estimate the α -stable distribution parameters in all the VLBI derived BLs considered in the present study. Table 3 contains the α -stable distribution parameters of the noisy independent components (IMF₁) across all the BLs. Though the α -stable distribution parameters vary across all BLs as per Table 3, all their values lie within the limits

of the α -stable distribution and are therefore consistent. Based on these results, it can be concluded that the independent noise components in all VLBI BLs could be described by the α -stable distribution.

Conclusion

Geodetic data archives have continued to increase owing to availability of data from various space geodetic techniques over the years. The available data have been utilised by the scientific community for various research applications. One area of research is in the analysis of the geophysical signal structure embedded in the geodetic data sets. Analyses of these data have however revealed that geodetic data sets often show excursions as well as self-similar or scaling properties. The observed unevenness could be associated to the independent statistical properties of the geophysical and noise components present in the data. One of the best approaches to explore the nature of individual components inherent in geodetic data is to model the statistical properties of the individual components.

One way to model the noise components embedded in geodetic data is by using the distribution function. In order to describe an appropriate distribution function of data, the first step is to assess the nature of the distribution function by use of proxy parameters. For instance, α -stable processes are known to exhibit scaling behaviour, are heavy tailed and are long-range dependent; all these statistical features have been observed in geodetic data and have been reported in the literature. As a result, using α -stable distribution to characterise the nature of the geophysical and noise components in geodetic data could be appropriate.

In the present contribution, the properties of α -stable distribution with long tails and infinite moments in geodetic data have been investigated by means of statistically deducing the underlying distribution. The main contribution of the current analysis in terms of geodetic data analysis is assessing whether the frequency distribution of the geophysical components in geodetic data is stable. If the stability is inferred, then the noise characteristics present in the data would be modelled. Our results generally demonstrate that the independent noise components in geodetic data are α -stable distributed.

Acknowledgements

This work was supported by the Inkaba yeAfrica project, A German-South African collaborative Earth Science initiative. The authors would like to thank the reviewers for constructive comments and useful suggestions that improved the manuscript; as well as the International VLBI Service (IVS) for providing high quality geodetic products. A number of Matlab scripts utilised in the analysis were the modified versions of the original and freely available EMD and FastICA software package. This is Inkaba yeAfrica contribution 58.

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