

# **The application of order tracking for vibration analysis of a varying speed rotor with a propagating transverse crack**

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## **Abstract**

Propagating cracked rotor vibrations may contain substantial information on the rotor crack conditions. Capturing the characteristic vibrations due to the non-stationary response of rotors of which the speed change, is useful for condition monitoring of such systems. From the literature it is clear that vibrations at harmonics of the rotational speed, as well as transient responses, may be considered key indicators of cracks in rotors. Since faults other than cracks, such as misalignment and imbalance also generate harmonic vibrations, the non-order related vibrations therefore become important characteristics in detecting cracked rotor problems. But these signals may present with small amplitude and may easily be missed among the non-stationary harmonic vibrations. Therefore, clear identification of harmonic as well as non-order related (transient) vibrations are both important for detecting cracks in rotor systems. In this paper, a finite element model is used to calculate the response of a rotor with a propagating transverse crack under varying rotational speed conditions. Various order tracking techniques, i.e. computed order tracking, Vold-Kalman filter order tracking, Gabor order tracking, are implemented to remove the varying speed harmonic vibrations so that the non-order related vibrations in which information about the cracks are contained, are emphasized. And some other signal processing methods that may achieve similar effects, i.e. double re-sampling method and intrinsic mode function from empirical mode decomposition, are also discussed for comparisons to these order tracking techniques. The paper demonstrates the advantages of order tracking methods in rotor crack detection.

**Keywords: Crack detection, Computed order tracking, Gabor Order tracking, Varying rotational speed, Vold-Kalman filter order tracking.**

## 1. Introduction

With the ever growing demands of industry, rotating machineries are operating at higher speeds, higher power and increasing load conditions. Shaft crack problems may therefore present significant hazards in rotating machines, especially turbo machinery. This may even lead to catastrophic failure of systems [1]. Early detection of shaft cracks is therefore of significant importance to the reliability, availability, efficiency and safety of rotating machinery.

Cracks destroy the symmetry of a rotor and introduce local flexibility. The local flexibility of a rotating shaft is essentially constant for a gaping crack. However it is not constant for a breathing crack, due to the opening and closing mechanism [2]. Clearly, this kind of local flexibility in the structure will influence the dynamic behavior of a rotating system and behaves like external excitation forces acting on the system [3]. Researchers have used simple rotor models to study cracked rotor dynamics since the 1970s (e.g. [4] and [5]). More advanced cracked rotor models have subsequently been proposed and are used to investigate dynamics of crack behavior (see for instance [2], [6] and [7]). Normally variation of stiffness and damping matrices is introduced into the equations of motion of the system to simulate cracks in a rotor shaft. These variations of stiffness and damping matrices represent the influence of a crack on the dynamic properties of the rotor shaft.

Owing to insufficient knowledge of the true stiffness and damping variations associated with cracks, it is very difficult to obtain accurate mathematical models. However, despite the limitations of these models, typical dynamic responses which are to be expected from cracked rotor systems have been reported in the literature (such as [5], [8] and [9]). This has created sufficient understanding of the nature of cracked rotor response, to pave the way for using signal processing methods to extract characteristic vibration signals of cracked rotors for diagnostic purposes. The advantage of using a data-based signal processing approach for the detection of cracked rotors stems from the fact that it does not require much knowledge about the system, such as the mathematical model or modal properties. Only knowledge about the characteristic nature of the vibrations is required [3].

In 1998 Zheng [3] synthesized previous research results of cracked rotor dynamic response and summarized the mathematical models for typical vibration responses generated by a fatigue crack. He showed that vibration signals from a cracked rotor could possibly be distinguished from other kinds of vibration signals by three features: extra transient vibration components, at least two impulse-like excitations per rotation and chaotic vibrations. Among these features, one of the well recognized characteristic responses of a cracked rotor is the increase of the second harmonic component of the rotational speed, which has been widely reported in various researches (such as [4], [9] and [10]). Unfortunately these kinds of harmonic responses can also be generated by other rotor related machine problems, such as

rotor misalignment, imbalance, etc. It is therefore not always appropriate to choose the response at harmonics of certain rotational speeds as major features for the detection of cracks in rotor shafts.

A more practical and distinguishable approach to detecting cracks in rotors may be to detect transient vibrations by signal processing methods [3]. However, vibration signals of rotor systems are generally dominated by harmonics of the rotational speed. The characteristic transient vibration signals for cracked rotors may therefore often be masked by the dominant harmonic vibrations and may easily be missed. Identifying the trace of these vibrations will therefore definitely help the analyst to clearly understand the condition of the cracked rotor and distinguish rotor crack problems from other rotor related problems. In the literature, research on cracked rotor transient vibrations is mainly focused on their dynamic behavior (e.g. [11] and [12]). However, little has been published on discriminating between transient vibrations and dominant harmonic vibrations. In this regard Zheng [3] suggests using the Gabor transform to remove the harmonic components from the cracked rotor responses and detect the transient vibrations. Although Zheng's work is effective in removing harmonic content and renders useful transient vibrations for crack detection, it is however based upon the assumption of stationary rotational speed, and does not present the ability of adaptively capturing those transient vibrations under continuously changing rotational speed conditions.

In this paper the adaptive natured time waveform reconstructed order tracking method, Vold-Kalman filter order tracking (VKF-OT), is employed in the application of obtaining characteristic cracked rotor vibrations through which frequency varying harmonic vibrations can be traced and further removed from the propagating cracked rotor responses. So that the ever changing rotational speed harmonics and their related transient vibrations for cracked rotor responses could be separated. The similar Gabor order tracking(GOT), is also discussed and compared with Vold-Kalman filter order tracking in the removal of varying harmonic vibrations. Moreover, computed order tracking (COT) is also applied to assist the process of order removal as well as interpretation of the separated cracked rotor response. All these open opportunities to enhance the understanding of cracked rotor vibrations. Furthermore, some other similar order removal techniques, i.e. double re-sampling method and Intrinsic Mode Function(IMF) from Empirical Mode Decomposition(EMD), are also discussed for comparisons. From comparisons, it proves that the application of order tracking methods offers a better solution to crack detection. In the first part of the paper, the nature of cracked rotor dynamic responses as well as the use of order tracking methods for cracked rotor vibrations is discussed. In the second part, the VKF-OT, GOT and COT are applied to a finite element rotor model with a propagating transverse crack, to demonstrate the ability of these order tracking methods to identify characteristic crack vibrations.

## 2. Theoretical basis

### 2.1 A cracked rotor response model

It is useful to consider a typical mathematical expression for cracked rotor response from the literature, so that a clear understanding on the nature of cracked rotor response may be obtained. Nayfeh and Mook [8] present an approximate solution for cracked rotor response  $x(t)$ , assuming very small cracks:

$$x(t) = \left\{ \sum_{i=1} U_i e^{\lambda_i t} \right\} + \sum_{k=1} \sum_{i=1} \frac{V_{ik}}{(\lambda_i - jk\Omega)} e^{jk\Omega t} + \sum_{\substack{m=1, n=1 \\ m \neq n}} \frac{W_{mnk}}{(\lambda_m + \lambda_n - jk\Omega)} e^{jk\Omega t} + E \quad (1)$$

Here  $U_i$  is the coefficient of the transient part  $\left\{ \sum_{i=1} U_i e^{\lambda_i t} \right\}$  of the vibration response and is determined by the system characteristics (mass, stiffness and damping) and the initial conditions of the crack, while it is not directly determined by the rotational speed of the shaft  $\Omega$ .  $\lambda_i = \zeta_i + j\omega_i$  where  $\zeta_i$  is the damping coefficient and  $\omega_i$  is the natural frequency of the system.

$V_{ik}$  and  $W_{mnk}$  are coefficients which are determined by the system characteristics and the initial conditions at the instant that the crack begins to open.  $i, k, m, n = 1, 2, 3, \dots$ , and  $E$  represents the complex conjugate part of the solution. Despite the complicated appearance, the terms

$$\sum_{k=1} \sum_{i=1} \frac{V_{ik}}{(\lambda_i - jk\Omega)} e^{jk\Omega t} + \sum_{\substack{m=1, n=1 \\ m \neq n}} \frac{W_{mnk}}{(\lambda_m + \lambda_n - jk\Omega)} e^{jk\Omega t},$$

fundamentally represent orders based upon the rotational speed.

It is therefore clear that the response of a cracked rotor comprises a combination of transient vibrations and rotational speed harmonic vibrations or orders. The transient vibrations are not directly related to the rotational speed in this expression. However, it is known that they occur during crack breathing and crack breathing is related to the rotor revolutions. Therefore the transient vibrations are dependent upon crack behavior during rotor revolutions.

Other cracked rotor models are also reported in the literature. In the context of this work the important common feature of these models is that the cracked rotor response

comprises two parts: transient vibrations and rotational speed harmonic vibrations.

## **2.2 Using time waveform reconstructed order extraction and COT for cracked rotor vibrations under varying speed conditions**

As is mentioned above, transient vibrations from a cracked rotor are not linked to the rotational speed through a simple mathematical relationship, but are dependent on the crack behavior. The transient vibrations modulate the dominant harmonic vibrations, so that these order related vibrations are distorted from their original form. But the transient vibrations are usually small in amplitude compared with the dominant harmonic vibrations. Furthermore, with the variation of rotational speed, the transient vibrations become even more difficult to identify. This behavior can therefore not easily be extracted through time waveform reconstructed order tracking methods (the term, time waveform reconstructed order tracking, is mentioned by [13]) such as Vold-Kalman filter order tracking (VKF-OT) or Gabor order tracking (GOT), which are based upon rotational speed. Although, Gabor order tracking could be implemented with the absence of rotational speed, it still needs to construct a proper mask operation matrix[14] so as to reconstruct transient vibration signals, however the unpredictable nature of transient vibrations is a substantial barrier for the determination of mask operation matrix for Gabor order tracking. Therefore, the direct extraction of transient vibrations from original measured data is not easy to achieve for order tracking techniques. For this reason, with the help of definite information of rotational speed, VKF-OT or GOT can be used to remove the rotational speed harmonic vibrations from the data, so that the residue signals may render the non-order related vibrations in which the transient vibrations are contained. However, to determine which order(s) need to be subtracted from the original data is a must for the process of rotational speed harmonic removal.

To deal with this, COT may first be applied to establish the order content of the rotor vibrations. COT, to some extent, is one of the most simple and clear techniques to figure out the order content of the measured data. Time-frequency domain analysis as a substitute can also be used to determine order(s) of interest. However, the analysis of the data still may remain stationary assumption, such as Short Time Fourier transform, which violates the nature of the data when speed is varying. This will further discussed in the following simulation studies. Once the rotational speed harmonics are determined, VKF-OT or GOT can subsequently be applied and the harmonic vibrations can be removed from the rotor response. This renders resultant residue signals which contain the transient vibrations. Non-stationary effects due to the rotational speed changes however also remain in these residues. Thus COT may again be applied to the residue signals to render an order spectrum which excludes speed variation influences. So that the interpretation of transient vibrations from cracked rotor is realized in term of orders through the use of different order tracking analysis.

The application of order tracking techniques in this way achieves the decomposition of cracked rotor vibrations and excludes the effects of speed variation to improve the understanding of the rotor crack condition. Wang and Heyns [15] recently proposed the combined use of VKF-OT and COT in sequence, where the enhanced Fourier analysis order components are presented. However, the order tracking application in this paper is different from the sequential use of the two methods considered in [15], but rather uses each technique on its own for different purposes. A graphic explanation of the above discussion is given in Figure 1.

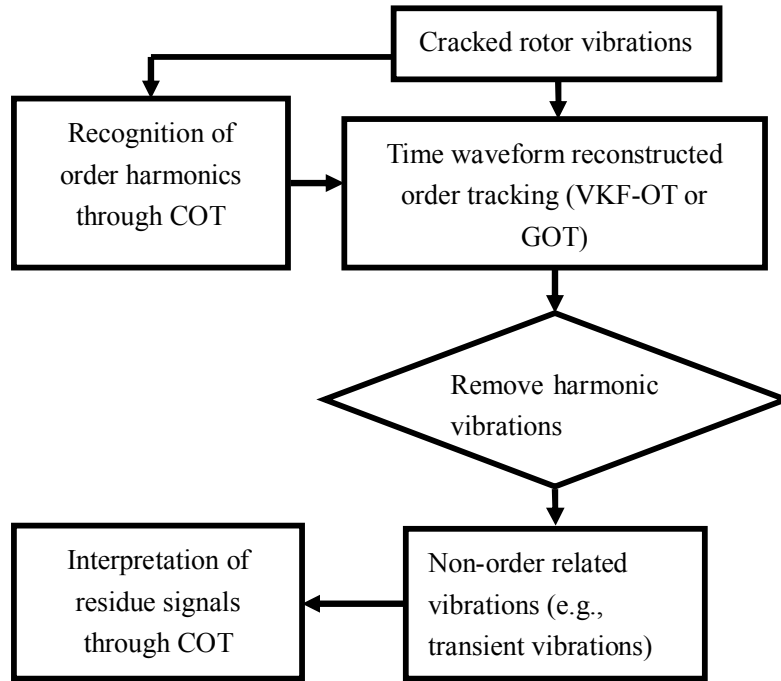


Figure 1: Use of COT and time waveform reconstructed order tracking(VKF-OT or GOT) on cracked rotor vibrations

### 3. Cracked rotor model

A FEM model of a cracked rotor which was developed by Guo and Peng [10] is used here. A brief description of the model and the equation of motion is given below.

A cracked shaft is loaded by three tensional and three torsional forces. The equation of motion of the cracked rotor system is given by

$$[M]\{\ddot{q}\} + ([C] + \Omega[G])\{\dot{q}\} + f(\psi)[H^T]([K_0] - [K_c])[H]\{q\} = \{F\} \quad (2)$$

where  $[M]$  is the mass matrix,  $[C]$  is the bearing oil-film damping matrix,  $[G]$  is the gyroscopic matrix,  $[K_0]$  is the global stiffness matrix without a crack and  $[K_c]$  is the global stiffness reduction caused by the crack.  $[H]$  is the transformation

matrix between rotating co-ordinates and inertial co-ordinates,

$$[H] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \cos \alpha & -\sin \alpha & 0 & 0 & 0 & \dots \\ 0 & \sin \alpha & \cos \alpha & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \cos \alpha & -\sin \alpha & \dots \\ 0 & 0 & 0 & 0 & \sin \alpha & \cos \alpha & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where  $\alpha$  is the angle between the two co-ordinate systems.

$\{F\}$  is the excitation vector including the unbalance forces and gravity forces. The

local stiffness matrix of cracked element  $[K]_{crack}$  can be written as

$$[K]_{crack} = [T]^T [c]^{-1} [T]$$

where

$$[T] = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -l & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & l & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$l$  is the length of rotor

The flexibility matrix of a cracked element can be expressed as

$$[c] = [c]^{(0)} + [c]^{(1)}$$

where  $[c]^{(0)}$  is the flexibility matrix of an element without a crack and  $[c]^{(1)}$  is the additional flexibility matrix due to the crack, which has

$$[c]^{(1)} = \begin{bmatrix} c_{11} & 0 & 0 & 0 & c_{15} & c_{16} \\ 0 & c_{22} & 0 & c_{24} & 0 & 0 \\ 0 & 0 & c_{33} & c_{34} & 0 & 0 \\ 0 & c_{42} & c_{43} & c_{44} & 0 & 0 \\ c_{51} & 0 & 0 & 0 & c_{55} & c_{56} \\ c_{61} & 0 & 0 & 0 & c_{65} & c_{66} \end{bmatrix}$$

The expressions for  $c_{ij}$  can be found in [10].

The switch function which is used to simulate the opening and closing mechanism of the crack is expressed as

$$f(\psi) = \frac{1 - \cos(\psi)}{2} \quad (3)$$

where  $\psi = \omega t - \theta + \pi$  and  $\theta$  is related to the curvature of the shaft at the crack location and determined by dynamic response of the shaft. The angle  $\theta$  is shown in Figure 2 and is the angle between the  $x$  axis and a line between the rotor axis center  $O_2$  and origin  $O$ .

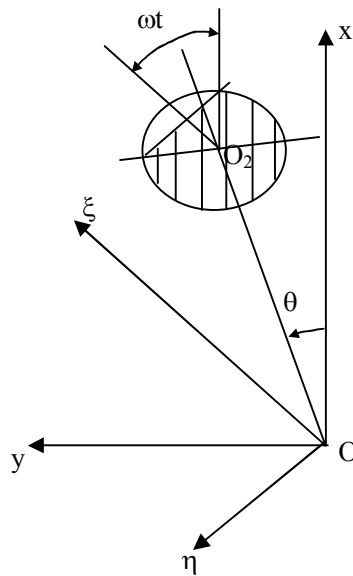


Figure 2: Switch function angles

## 4. Processing of the simulated cracked rotor response

### 4.1 Traditional signal processing for simulated cracked rotor response

The simulation considers an accelerating rotor with conditions as follows:

- 1) the angular acceleration is 10 Hz per second,
- 2) the depth of crack propagates from 0.05 mm to 3 mm over the simulation period,



3) and the crack propagation rate is 0.01 mm per rotation.

Vibration signals before the crack depth reaches 3mm (the first 8 seconds of the response) is considered. Figure 3 presents an analysis of the propagating cracked rotor response signals (acceleration in the  $x$ -direction) through traditional signal processing methods.

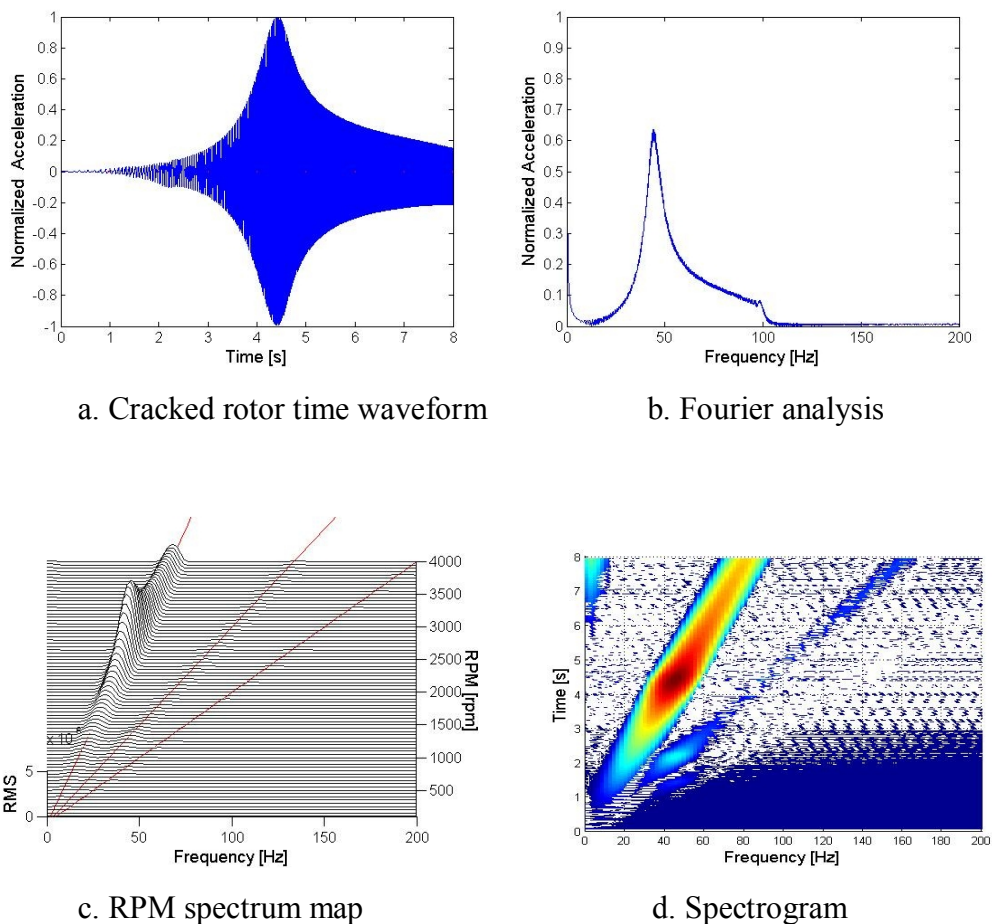


Figure 3: Cracked rotor acceleration in the  $x$ -direction

Figure 3(a) is the time waveform (the signal is being normalized in terms of highest amplitude and this normalization process will be also implemented in the following). A clear resonance hump can be seen at about 4.5s. This corresponds to Figure 3(d) where the spectrogram also exhibits the same result at that time. The spectrogram further shows that this resonance becomes severe on the 1<sup>st</sup> rotational speed harmonic signals or 1<sup>st</sup> order and that this 1<sup>st</sup> order of the signal dominates the whole spectrum. Figure 3(b) is the Fourier analysis result of the original signal. The highest frequency peak is located just below 50 Hz. However, note that there is a wide range frequency distribution around 50 Hz. This is due to the angular acceleration of the rotor. The effect can further be observed in Figure 3(c) in which the RMS value of the 1<sup>st</sup> order signal rises to the maximum again at about 50 Hz with the corresponding rotational

speed of 3000 rpm. Furthermore, from both Figures 3(c) and (d), the 2<sup>nd</sup> and 3<sup>rd</sup> order signals, albeit small, can also be observed. In Figure 3(c), the 1<sup>st</sup> to 3<sup>rd</sup> order lines are also drawn for illustrative purposes. For cracked rotor response, harmonic order signals, especially the 2<sup>nd</sup> harmonic is known to be important for crack detection. The above traditional signal processing methods corroborate the fact that the simulated responses represent a cracked rotor. However none of these analyses, which fundamentally assume steady conditions, provide clear information about the transient vibrations. Thus, order tracking methods are applied to further clarify the composition of the cracked rotor response.

## 4.2 Application of COT and VKF-OT

### 4.2.1 Recognition of harmonic vibrations through COT

In order to follow the detailed progression of the cracked rotor vibration in terms of rotational speed over the 8s, the simulated rotor vibration is divided equally into eight 1s periods for analysis, and computed order tracking(COT) is used to plot order domain spectral maps in Figure 4(a). The use of computed order tracking features a unique ability to obtain a clear overview of order contents of the signal which is not achievable by traditional Fourier transform, as for comparison, the Fourier spectral maps are also plotted in Figure 4(b) where the smearing effects due to the variation of rotational speed are clear and pose difficulties for recognizing the order contents of the signal especially at the first two second. For visualization purposes all figures are normalized in terms of the highest order or frequency peak in each period.

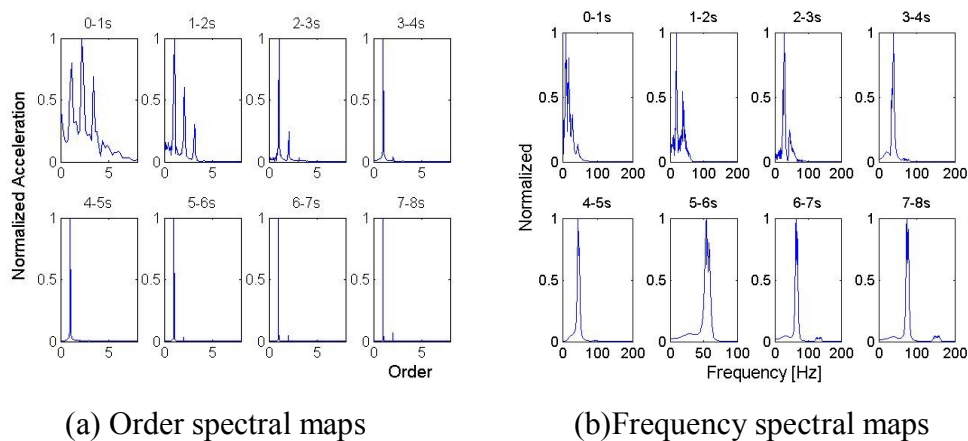


Figure 4: Order and frequency spectra of rotor vibration over successive 1s periods

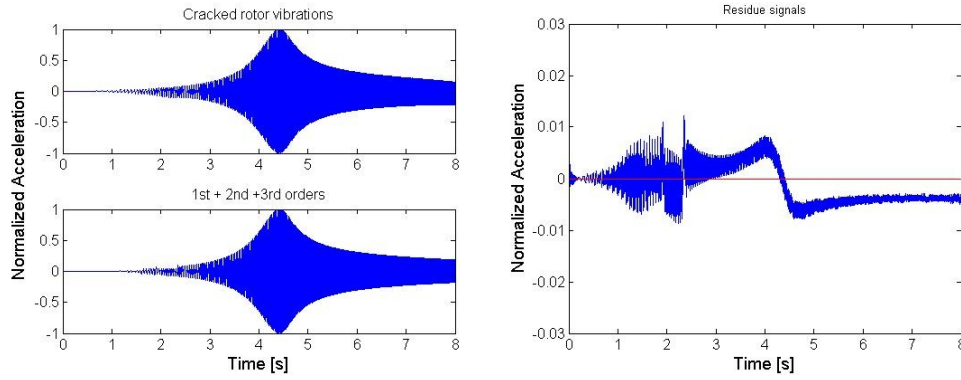
Figure 4(a) describes the development of the rotational speed harmonic content with time. It shows that during the first three seconds, the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> rotational speed harmonics are dominant (the first three order spectra in Figure 4(a)). These conclusions, however, cannot be easily obtained from Figure 4(b). This features the advantages of COT over Fourier transform in order recognition. When approaching system resonance from 4 to 5s, even the 2<sup>nd</sup> harmonic becomes negligibly small

compared to the dominant 1<sup>st</sup> order. Once the system resonance is passed during the period from 5 to 8s, the 2<sup>nd</sup> harmonic becomes evident again and increases compared with the dominant 1<sup>st</sup> order. This effect can be seen in both Figure 4(a) and (b). The comparison of Figure 4(a) and (b) clearly tells that, at this stage, order spectral maps provide a better representation of order contents of the signal and it offers a clear clue for the following removal of harmonic order vibrations. Moreover, it should emphasize that COT analysis, to some extent, is indispensable to the subsequent application of harmonic order(s) removal. Although, time-frequency domain analysis, such as Figure 3 (c) and (d), can capture the variation of the data so that the order content could be estimated, however it does not link rotational speed tightly and signals are plotted in three dimensions which brings some ambiguities for order recognition. For example, in Figure 3(c) and (d) the 3<sup>rd</sup> harmonic order is barely discernable. Therefore, for the sake of order recognition, COT method is a better choice.

To summarize, it is clear that the 1<sup>st</sup> harmonic of rotational speed vibration is dominant throughout the period of crack progression. The 2<sup>nd</sup> and 3<sup>rd</sup> harmonics are characteristic responses due to the crack but they become small compared with the dominant 1<sup>st</sup> harmonic when passing through the system resonance. However in general, the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> harmonics are major harmonic components and dominant in the order spectra. Again, it should emphasize that computed order tracking(COT) is essential to obtain these conclusions and it provides possibility for further signal analysis based upon specific orders. The non-order related vibrations, such as transient vibrations, however clearly are not prominent among these dominant harmonics in the order as well as frequency spectral maps. While the COT analysis fails to give a proper indication of the transient vibrations present in the predominantly harmonic vibration environment, it does provide a clear idea of the harmonic content in the data. The removal of harmonic order vibrations is then applied in the following to render the non-order related vibrations.

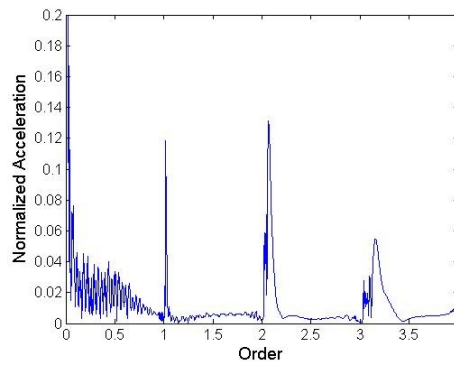
#### **4.2.2 Application of harmonic order removal by VKF-OT**

The VKF-OT method, in this paper, is used as the demonstration method for order removal. Some other order removal methods are also discussed in the following. To begin with, the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> harmonic vibrations are therefore now removed from the rotor vibrations by VKF-OT. The original rotor vibration and the sum of first three orders are shown in Figure 5(a). Clearly, the VKF-OT for harmonic vibration filtering accounts for most of the energy of the data.



a. Crack rotor response and its harmonics

b. Residue signals through the subtraction of 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> orders by VKF-OT



c. Order domain result of residue signals

Figure 5: Application of VKF-OT and COT

Figure 5(b) shows the resultant residue signal when the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> rotational speed harmonic components are removed from the rotor vibration. A 60% Vold-Kalman filter bandwidth is used for the 1<sup>st</sup> harmonic filtering, while a 20% Vold-Kalman filter bandwidth is used for the 2<sup>nd</sup> and 3<sup>rd</sup> harmonics. (The 60% and 20% relative filter bandwidth means that the ratio of the instantaneous absolute filter bandwidth to the instantaneous rotational speed frequency is 0.6 and 0.2. A relatively wider filter bandwidth of 60% is used in filtering for 1<sup>st</sup> rotational speed harmonic, because it is the dominant order and when passing through the system resonance, a wider filter bandwidth will ensure lower error in the amplitude over the resonance period. For details on the choice of the filter bandwidth, the reader may refer to [16].)

Under the non-cracked conditions of such an ideal rotor model, the rotor will only yield harmonic vibrations. However by the removal of the harmonic vibrations from the simulated cracked rotor vibrations, some new information is rendered. From the literature cited in paragraphs 1 and 2.1, it is known that this information describes the transient vibrations caused by the propagating crack. Although compared to the dominant harmonic vibrations these vibrations are quite small, it is clear that the time domain information has been separated from the harmonic content and can now be

studied in isolation.

Firstly, unlike rotational speed harmonic vibrations that are basically zero mean waveforms, as is shown in Figure 5(a), the amplitude of the residue signals fluctuates from positive to negative values when the system passes through resonance, in other words, a phase shift of the residue signal occurs (a zero line is drawn in the figure to display this shift). These results confirm observations in [12] that the transient vibrations vary when the system passes through the critical speed. This can be understood from figure 2.  $\theta$  indicates the angle of rotor real axis to the measured response direction, and this changes when the system passes through the critical speed. Changes in  $\theta$  lead to changes of the switch function that represents the cracked rotor behavior (see equation (3)). The significant increase of the system response when it passes through the critical speed has a large impact on  $\theta$  and therefore results in large changes of the cracked rotor behavior that causes the significant changes in the transient vibrations. As a result, the residue signals, in which the transient vibrations are contained, change significantly when passing through the system resonance. This could however not be detected without subtracting the changing order related responses.

Secondly, it may be observed from Figure 5(b) that the amplitude of the residue signal is increasing over the initial first few seconds (0 - 3 s). However as the crack progresses further as the rotational speed increases, this amplitude range decreases. This is clear by comparing the amplitude range before (1 - 3 s) and after (5 - 8 s) the rotor passes through the critical speed. This argument is also presented by Guo and Peng [10] who indicate that the effect of transient response is obvious at low rotational speed and propagation of a crack will abate the effect of a crack on transient response.

#### **4.2.3 Application of COT on residue signals**

Though the time domain transient vibrations in Figure 5(b) are useful for providing crack information as has been discussed above, it still does not offer an easy and direct impression of the signal. Thus, COT is applied on the residue signal and depicted in Figure 5(c). Previously, it was observed from Figure 4 that the rotor vibrations are strongly dominated by the harmonic vibrations of the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> orders. The other orders can barely be seen. Figure 5(c) however indicates that the residue signal order spectra occur slightly higher than these orders and order spectra are spread in a wider order range, especially for the 2<sup>nd</sup> and 3<sup>rd</sup> order relevant residue signal order spectra. Fundamentally speaking, it is known that the transient vibrations relate to crack behaviour and crack behaviour is influenced by the revolutions of the rotor or the rotational speed. Therefore, the transient vibrations should be closely related to the harmonic vibrations. However due to the non-direct relation to the rotational speed, the transient vibrations will not strictly coincide with the precise orders in the order spectra. Thus, the order domain analysis, further confirms the

nature of cracked rotor in that the transient vibrations are related to the harmonic orders but not exactly the same. More importantly, it is evident that compared to the order spectra of the original rotor responses in Figure 4, excluding the dominant harmonic vibrations makes the transient vibration effects easily detectable in the order domain.

In short, from the above discussions, it is found that the applications of COT and VKF-OT present a better solution to the understanding of cracked rotor vibrations. It also demonstrates that order tracking methods are superior to traditional signal processing methods in that it reveals the characteristics of the transient vibrations. The simulation study of a propagating cracked rotor proves the ability of COT and VKF-OT to assist analysis of crack rotor conditions.

#### **4.2.4 Discussions on other order removal methods**

With regard to the removal of the order contents, there are also some other methods may achieve the similar goal. Gabor order tracking (GOT) is another well known method. The method constructs mask operation matrix in terms of rotational speed and Gabor coefficients that are related to harmonic order vibrations can be selected and being used to reconstruct time waveforms for specific order(s). The resultant time domain signals through Gabor order tracking are very similar to Vold-Kalman filtering. The key factors to the successful implementation of GOT are smoothing rotational speed curve, defining filtering bandwidth, mask operation matrix construction and reconstructing order component[14]. The first two factors are essentially the same to requirements of VKF-OT. The last two factors, however, determines the differences between the two methods. It should be noted that, by Gabor transform, the measured data has been represented in terms of Gabor coefficients. The main purpose of the mask operation matrix is to select those Gabor coefficients that are representing harmonic order(s). For this specific cracked rotor case, the removal of harmonic order(s) is of interest. Therefore, once harmonic order related Gabor coefficients are being chosen according to rotational speed, these Gabor coefficients can then be set to zero and the rest of Gabor coefficients could be used to reconstruct residue signals through the inverse operation. In essence, Gabor coefficients are basis for Gabor order tracking or order removal. However, the basis for VKF-OT is to construct a pass band filter and deal with measured data directly. For illustrating this difference, the order domain result of residue signal by Gabor order removal is plotted in Figure 6(a), (the mask matrix for order bandwidth is set to be 0.2, the order spectral of the resultant residue signal is realized by COT). Note that its counterpart for VKF-OT result is depicted in Figure 5(c).

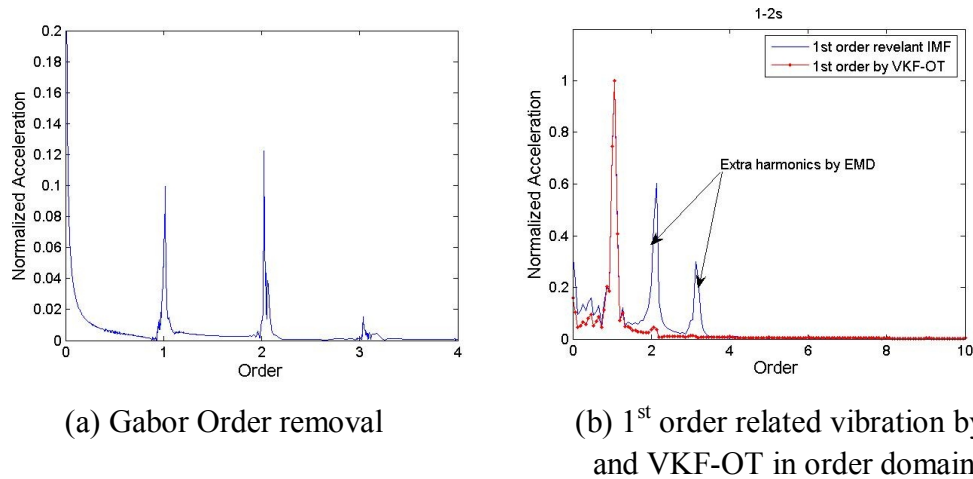


Figure 6: Application of Gabor and COT as well as comparisons between IMF and VKF-OT

By comparison, it may obtain that the previously mentioned order spread effects of 2<sup>nd</sup> and 3<sup>rd</sup> order in Figure 5(c) are also observed in Figure 6(a), which proves the effectiveness of Gabor order tracking method for crack detection. However, it also notices that by reconstructing Gabor coefficients instead of using filter to filter out the harmonic vibrations (i.e. VKF-OT), Figure 6(a) of order domain spectral appears low noise compared to Figure 5(c). But all in all, the two methods are both effective in order removal and crack detection. The characteristic transient vibrations are both presented in terms of order spectra.

There are also some other order removal methods. Groover et al.[17] utilize data re-sampling and transformation, between time, order and frequency domains to remove order components. The method is called double re-sampling and it is useful when harmonic order vibrations are being treated as corrupting phenomenon, such as structural resonance are buried in dominant harmonic vibrations for rotating machines. All the harmonic order related vibrations will be, theoretically, removed through the method. This is, however, not very suitable to the separation of transient vibrations for cracked rotor where transient vibrations are not fixed frequency component and close related to harmonic vibrations. Therefore, some of transient vibrations may also be removed through the method. Besides, the Groover's method has to deal with the whole measured data which in turn compromises the ability of focusing on specific order(s), such as 2<sup>nd</sup> or 3<sup>rd</sup> order in this cracked rotor case study. Thus, for the diagnostic analysis of cracked rotor vibrations, VKF-OT and GOT are more suitable for order removal to render transient vibrations.

Another possible order removal method is Intrinsic mode function (IMF) from Empirical Mode Decomposition (EMD). It has been widely studied in the field of condition monitoring rotating machines. Wang and Heyns [18], further interprets the relationship between IMF and order vibrations. In their work, an IMF has been

demonstrated as a special kind of order related vibrations. In fact, an IMF is very similar to the result of VKF-OT. Therefore, it is also worthwhile to discuss the order removal by IMF from EMD. Firstly, the name of Empirical Mode Decomposition (EMD) emphasizes that the method is empirical and no rotational speed is needed in the process of signal analysis. It infers that it is not based upon rigorous mathematics which is different from both VKF-OT or GOT. Secondly, Feldman [19] analyzed the special and useful case of the decomposition of two harmonics, demonstrating some of the important features of EMD, such as the nature of the resolution for each IMF. His results are further interpreted by Wang [20] in terms of order analysis that IMFs from EMD may include both orders and relevant vibrations and relevant vibrations usually are small in amplitude and close related to the dominant harmonic vibrations. In other words, IMFs from EMD may fail to separate order vibrations and its related vibrations. Therefore, IMF is also not an ideal method for order removal. For illustration, the EMD is applied to this cracked rotor vibrations. The 1<sup>st</sup> order relevant IMF in order domain as well as the 1<sup>st</sup> order spectral by VKF-OT is superimposed and normalized in Figure 6(b) (COT is used to obtain order spectra, the 1-2s signal is used for illustration). Figure 6(b) tells that the 1<sup>st</sup> order relevant IMF fails to separate 1<sup>st</sup> order and its related harmonic vibrations. Extra harmonics shown in Figure 6(b) is obvious and 1<sup>st</sup> order result from VKF-OT clearly with higher ability for discriminating of order(s). Therefore, with regard to order removal, IMF is naturally inferior to VKF-OT or GOT.

In summary, the abilities of the different signals processing methods in cracked rotor vibration detection are compared in table below.

Table 1: Comparisons of different kinds of methods' abilities on cracked rotor vibration detection

| Crack rotor vibrations        | VKF-OT or GOT method | COT  | Time-frequency analysis | Fourier analysis   | Combination of COT, VKF-OT or GOT. | Double re-sampling by Groover et al.[17] | IMF from EMD       |
|-------------------------------|----------------------|------|-------------------------|--------------------|------------------------------------|--|--------------------|
| Transient vibration detection | No                   | Weak | Weak                    | Weak               | Yes                                | No                                       | Weak               |
| Harmonic vibration detection  | Yes                  | Yes  | Yes, but not ideal      | Yes, but not ideal | Yes                                | Yes                                      | Yes, but not ideal |

## 5. Conclusion

The application of different order tracking analysis on cracked rotor vibrations enhances understanding of cracked rotor status under conditions of varying rotational speed. The transient vibration information which serves as a clear indicator of the



cracked rotor response can be separated out through the removal of harmonic vibrations and presented in the order domain. The ability of separating transient vibration information in a varying rotational speed environment improves the diagnostic capability. The comparisons of different signal processing methods prove that the application of different order tracking analysis is effective in rotor crack detection.

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